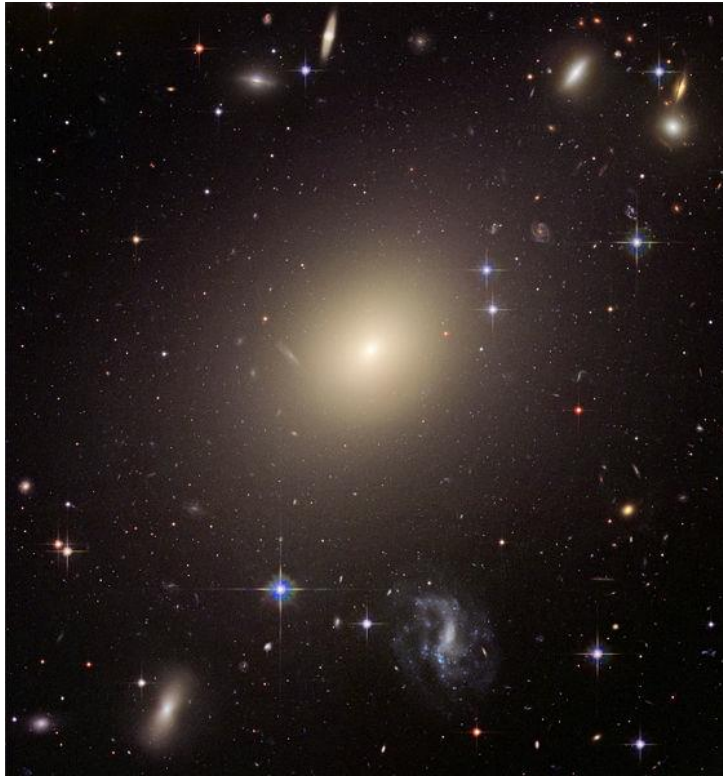
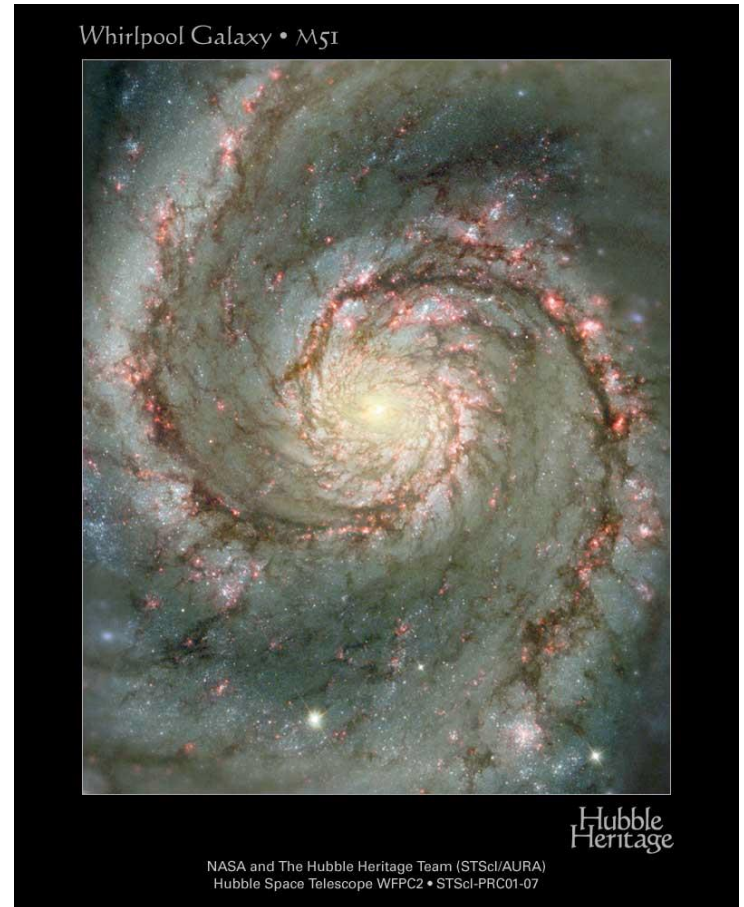


Galaxies



ESO 325-
G004



Early Sky Surveys

Messier Catalog

In the 18th century Messier recorded ~ 103 objects. Many of these are nebulae and some are galaxies.

M1: Crab SNR

M31: Andromeda galaxy

M45: Pleiades

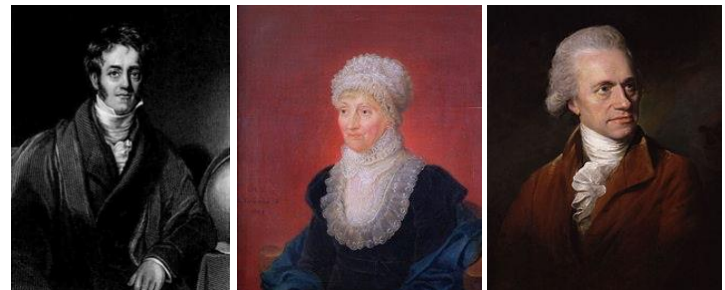
M42: Orion Nebula

New General and Index Catalogues

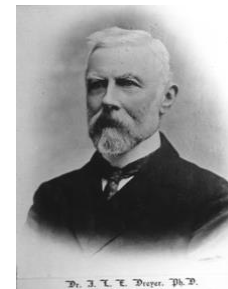
Compiled by John Louis Emil Dreyer that expand upon observations made by the Herschel's. Contains >10,000 objects, some inside and some outside our galaxy



Charles Messier



John, Caroline and William Herschel



John Louis Emil Dreyer

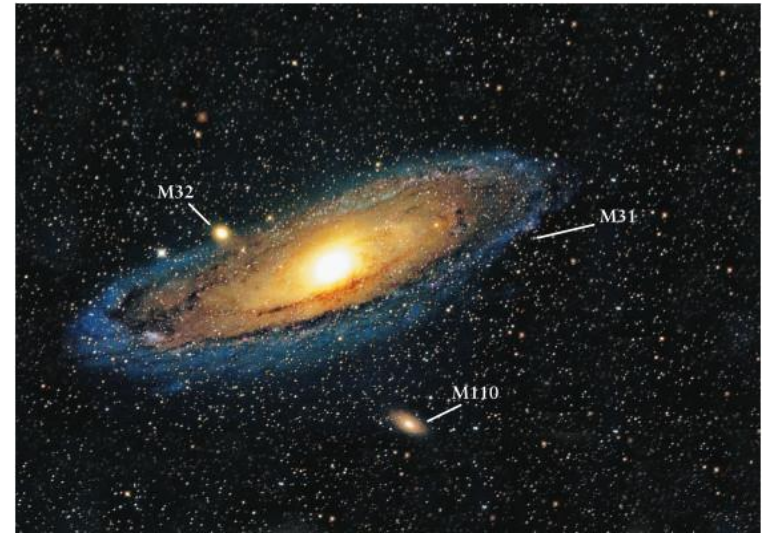
Galaxies

In the early 1900's astronomers were debating whether objects that appeared to be nebulae with spirals were:

(a) objects located within our galaxy or,

(b) rotating systems of stars just like our Milky way but not a component of our galaxy.

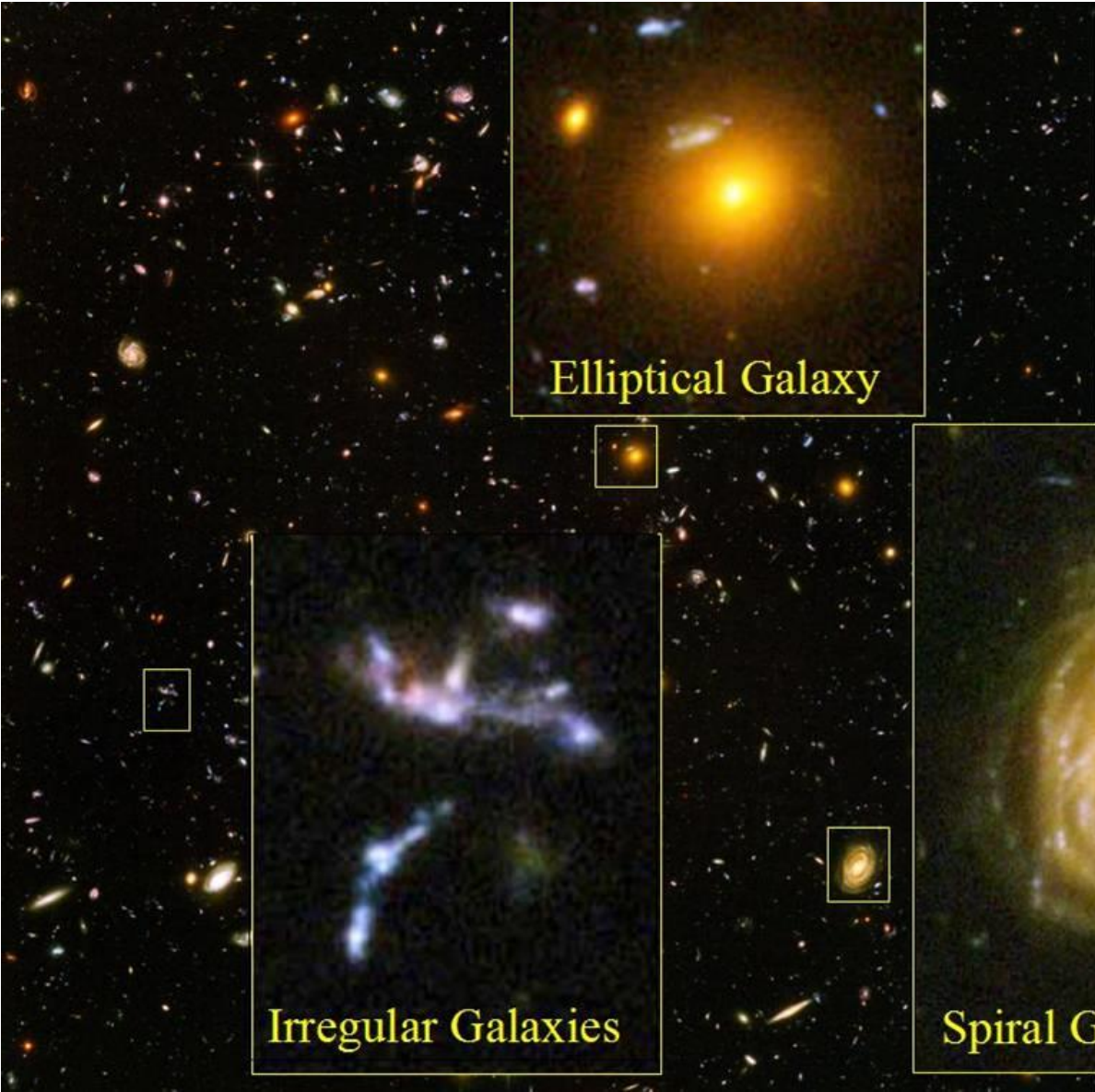
Harlow Shapley and Heber Curtis debated this issue in DC in 1920. The issue was not resolved at the time.



The nature of “spiral nebulae” was resolved with observations of Andromeda made by astronomer Edwin Hubble:

In 1923 Hubble measured the distance to M31 using a Cepheid variable star.

□ Hubble
Ultra Deep
Field



Elliptical Galaxy

Irregular Galaxies

Spiral Galaxy

Characteristics of Galaxies

Integrated Color

Ellipticals are redder (contain on average older stars) than spirals

Spectral Types in Galaxies

Ellipticals & Sa= mostly K stars (Pop I stars)

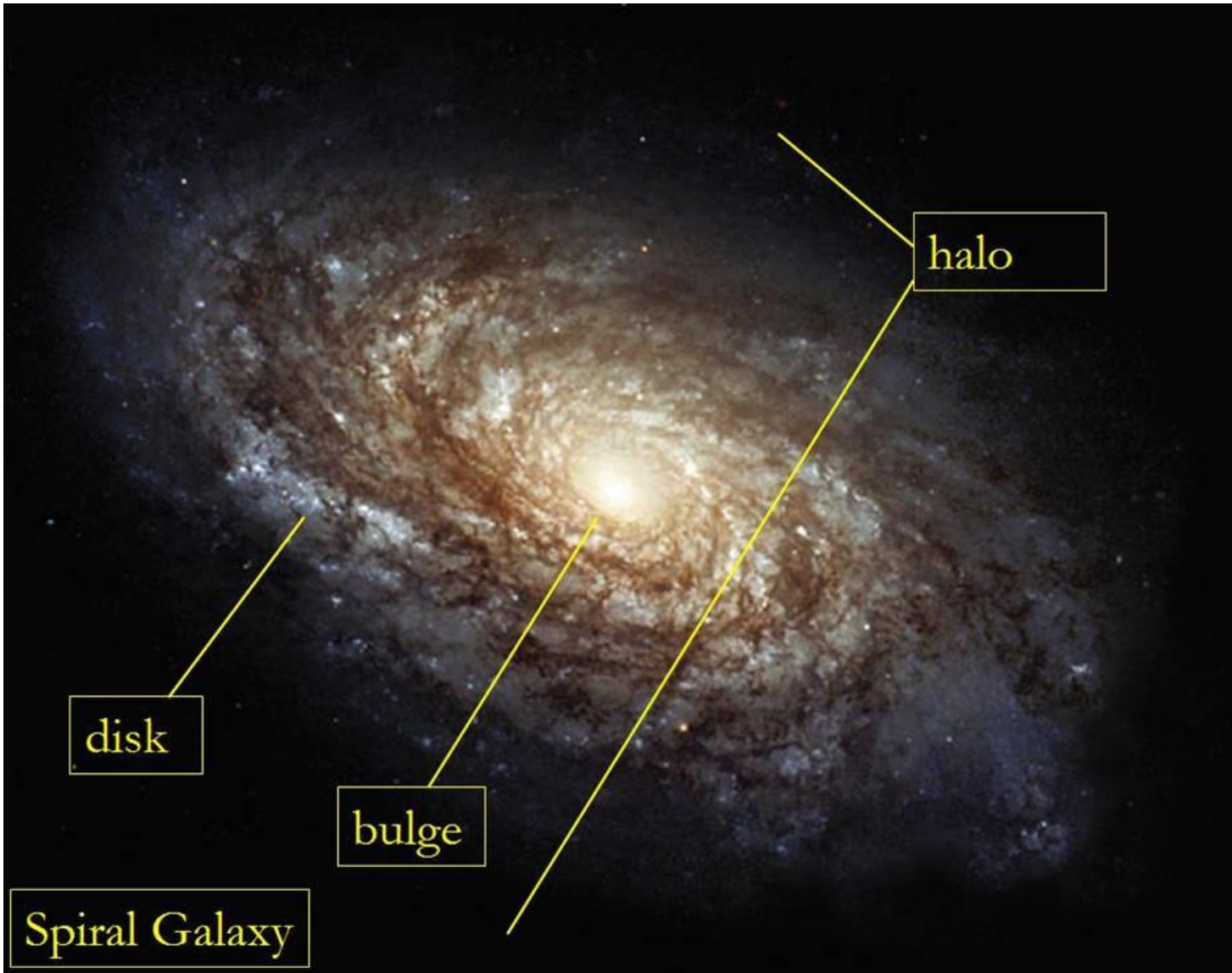
Sb: mostly F to K stars

Sc & Ir: A to F stars (Pop I)

Neutral Hydrogen Content

$\frac{M_{HI}}{M_{Galaxy}}$ = Ellipticals $\sim$ 0%, S0 $\sim$ 2%, Irregulars $\sim$ 22%

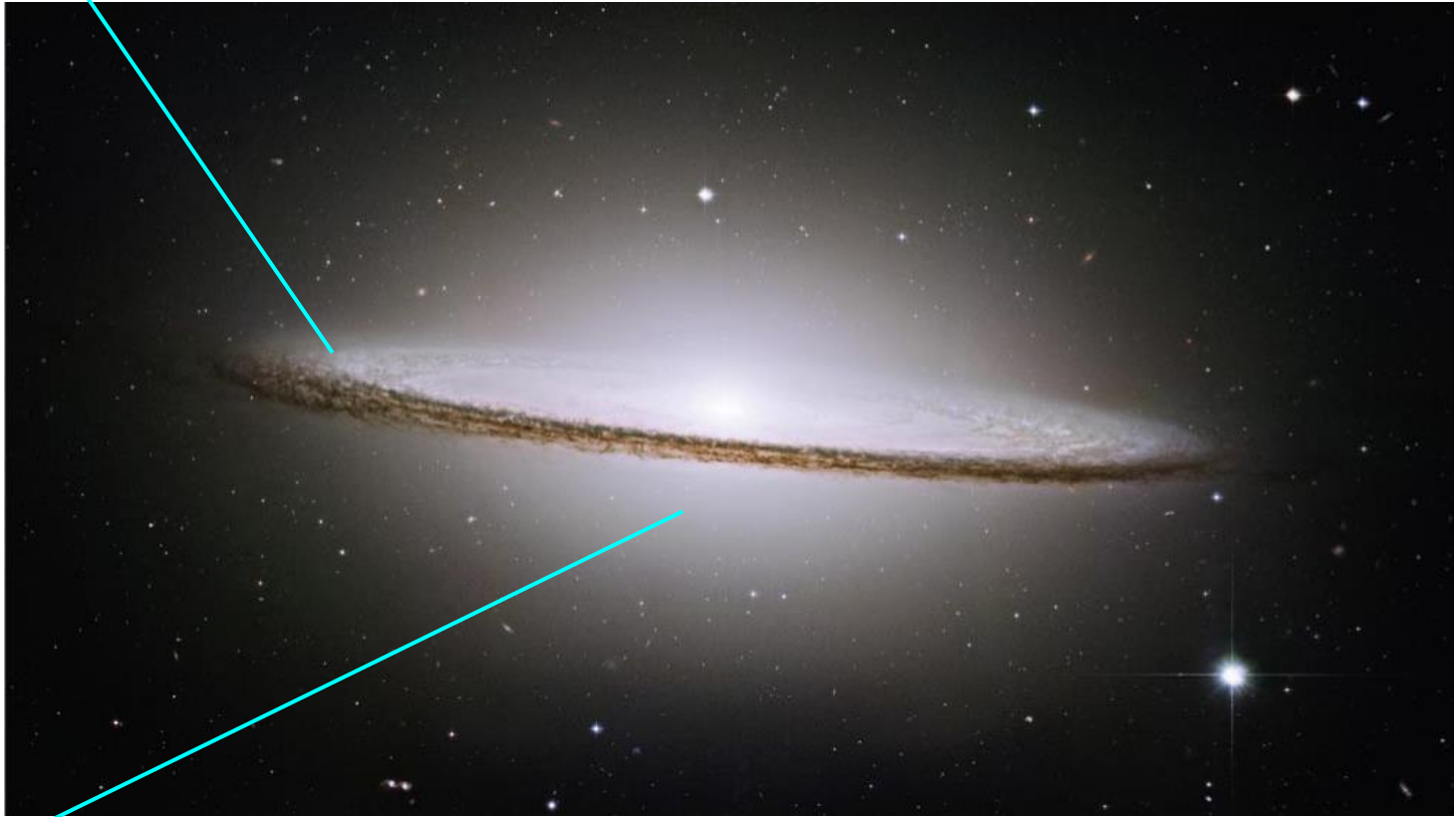
Spiral Galaxies



Spiral Galaxies

Disk component:

stars of all ages, many gas clouds



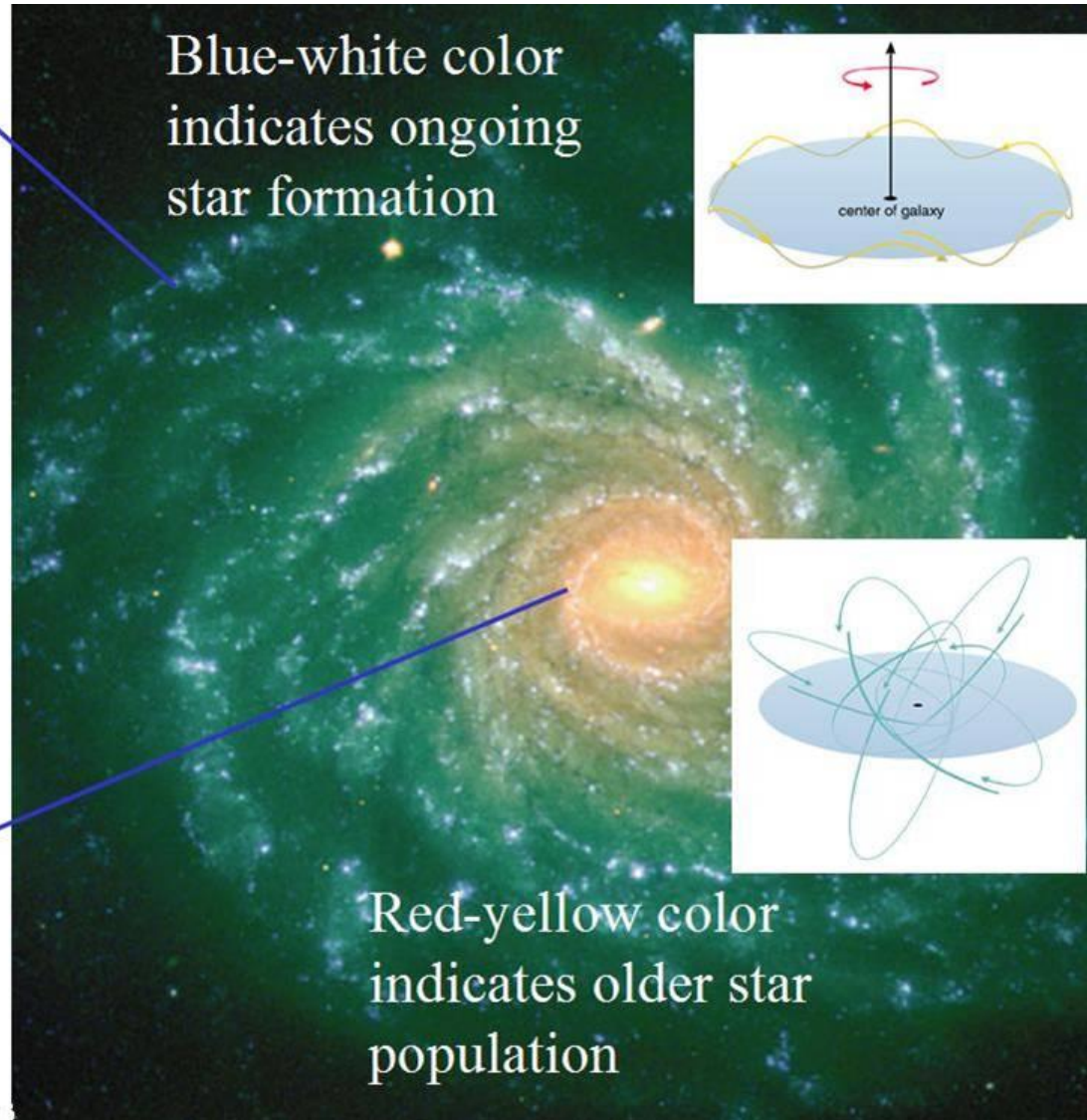
Spheroidal component:

bulge and halo, old stars in halo, few gas clouds

Spiral Galaxies

Disk component:
stars of all
ages, many
gas clouds

Spheroidal component:
Bulge (old and
young), halo
(old stars), few
gas clouds



Spiral Galaxies



(a) Sa (NGC 1357)



(b) Sb (M81)



(c) Sc (NGC 4321)

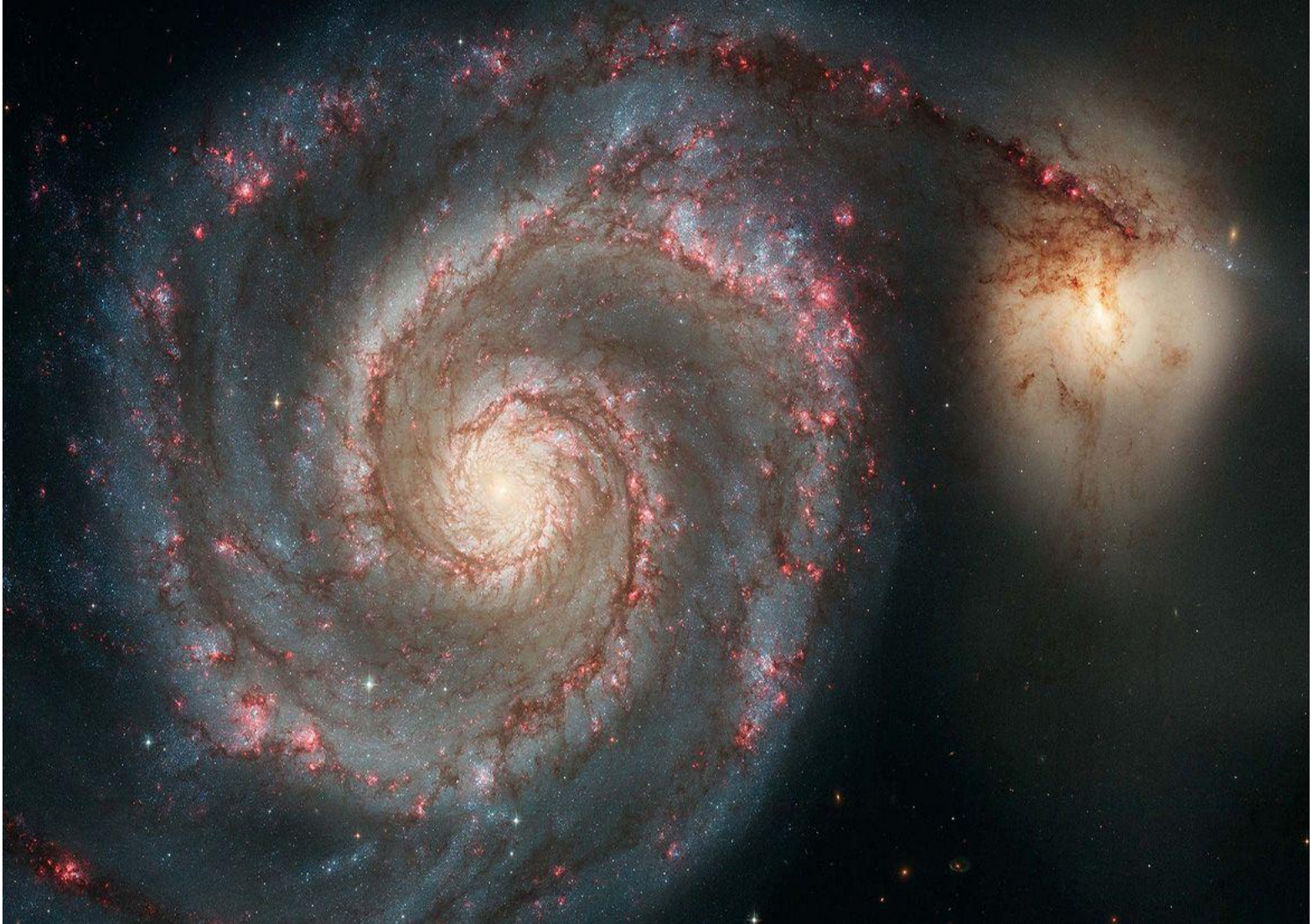
Hubble further classified spirals into :

Sa galaxies: smooth broad and tightly wrapped spiral arms with a large bulge ($\sim 4\%$ of baryonic mass in dust and gas)

Sb galaxies: moderate-sized spirals and bulge ($\sim 8\%$ of mass in dust and gas)

Sc galaxies: narrow loosely wrapped spirals and small bulge ($\sim 25\%$ of mass in dust and gas)

Grand Design Spiral



The Whirlpool Galaxy M51
Grand Design Spiral

Flocculent Spiral



Flocculent Spiral NGC 2841

Spiral Galaxies

10% are Grand Design
60% have Multiple Arms
30% are flocculent

Additional classifications to Hubble one made by de Vaucouleurs

cE E E⁺ S0⁻ S0⁰ S0⁺ S0/a Sa Sab Sb Sbc Sc Scd Sd Sdm Sm Im

Where **cE**= compact elliptical, **E** = normal elliptical **E⁺**= late type elliptical

Sm (SBm) - irregular in appearance; no bulge component

Im - highly irregular galaxy

Rings. Galaxies are divided into those possessing ring-like structures (denoted '(r)') and those without rings (denoted '(s)'). So-called 'transition' galaxies are given the symbol (rs).

The fraction of the stellar mass contained in a spheroid/bulge relative to the disk increases from right to left

Elliptical Galaxies



a M87, a giant elliptical galaxy in the Virgo Cluster, is one of the most massive galaxies in the universe. The region shown is more than 300,000 light-years across.

- **Elliptical galaxy:** all spheroidal component, virtually no disk component
- Red-yellow color indicates older star population.

Elliptical Galaxies

Elliptical galaxies contain very little amounts of gas, dust and star formation. They are composed of mostly old red Population II stars.

Giant elliptical galaxies are about 20 times larger than typical elliptical galaxies and often found in clusters of galaxies.

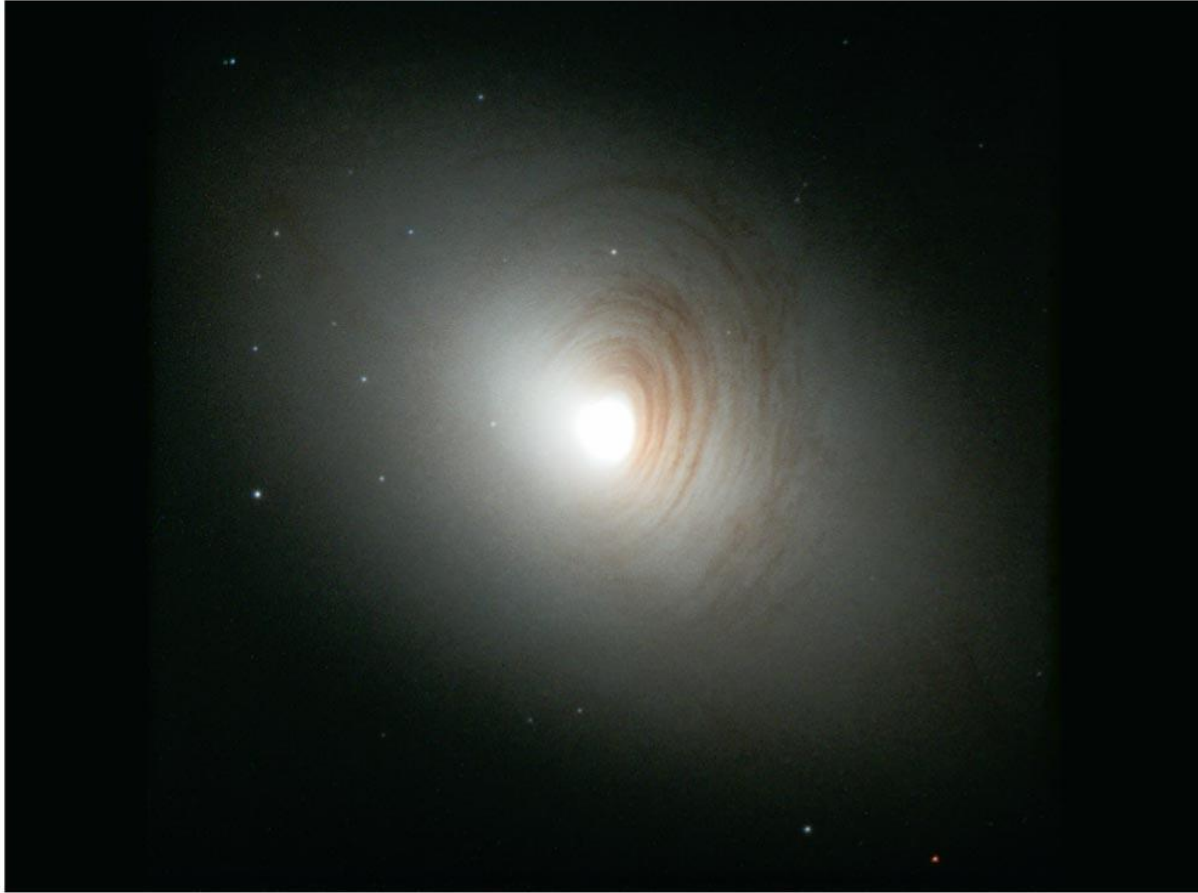
Dwarf elliptical galaxies contain only about a few million stars compared to 100 billion in the Milky way.



Blue-white color indicates ongoing star formation.

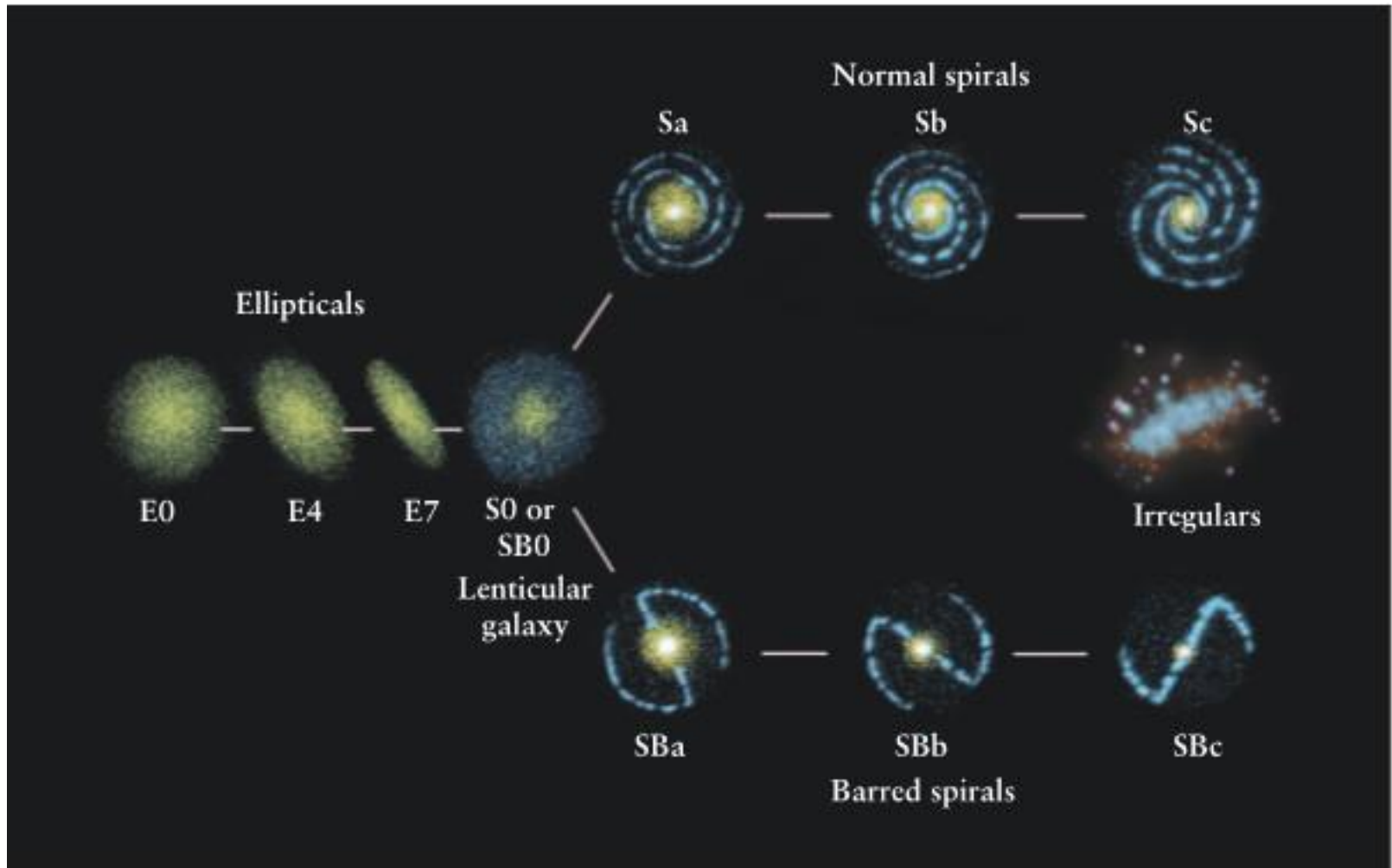
- ❑ **Irregular galaxy**(some have distorted shapes likely the result of collisions)

Lenticular Galaxies



- **Lenticular galaxy:** has a disk and bulge like a spiral galaxy but no spiral arms and has much less dusty gas (intermediate between spiral and elliptical)

Hubble's Tuning Fork Diagram

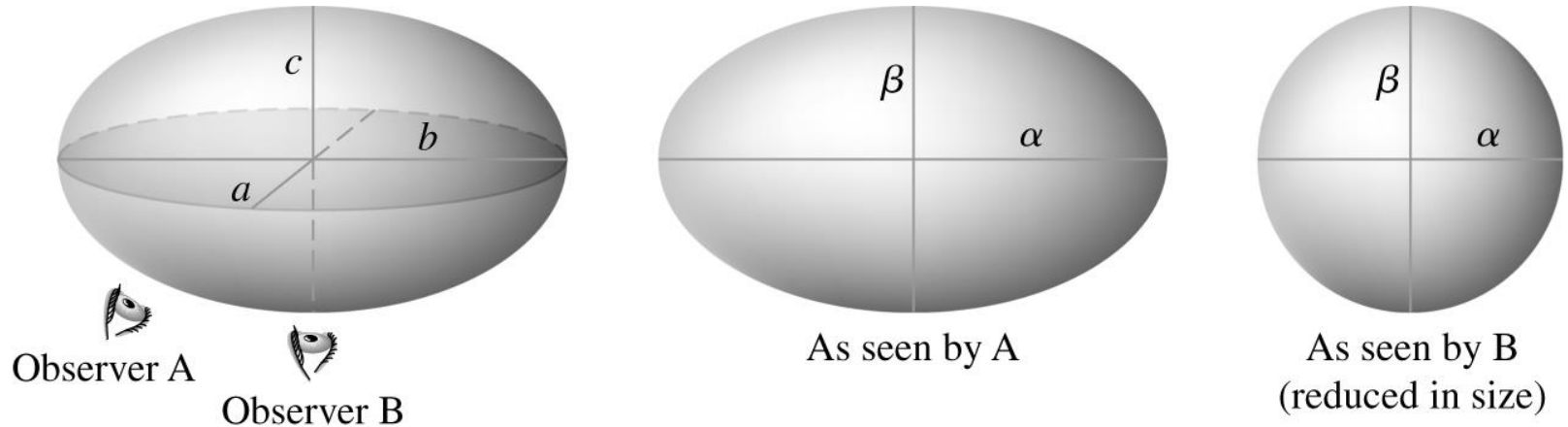


Spheroid
dominates

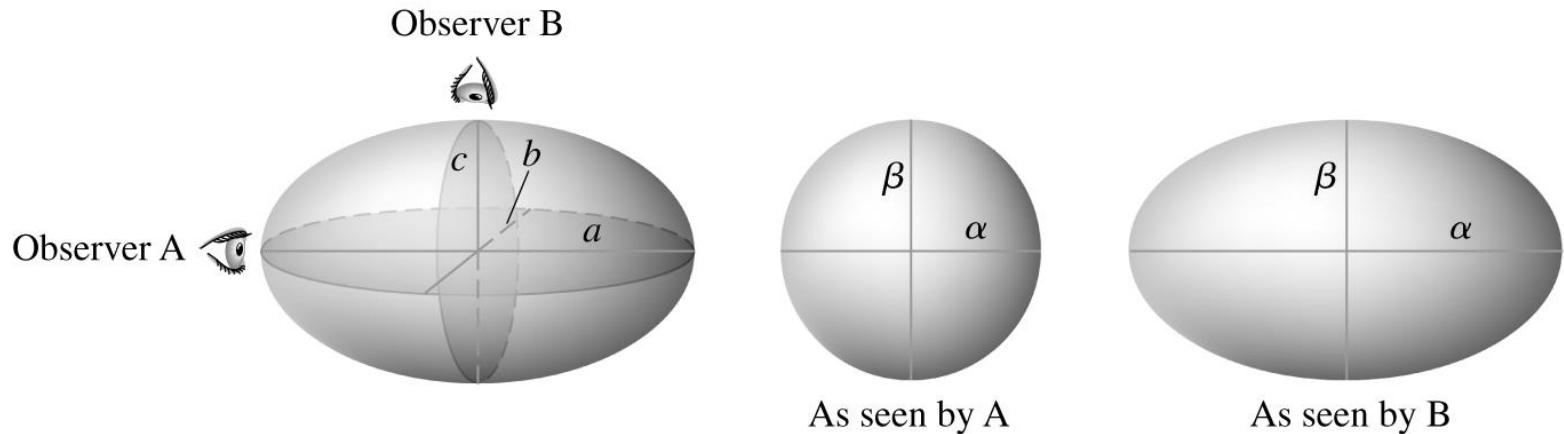
Disk
dominates

Ellipticals: Morphological Classification: E0 – E7

EX, with $X = 10(1-\beta/\alpha)$



oblate: flattened spheroid



Prolate: Football shaped

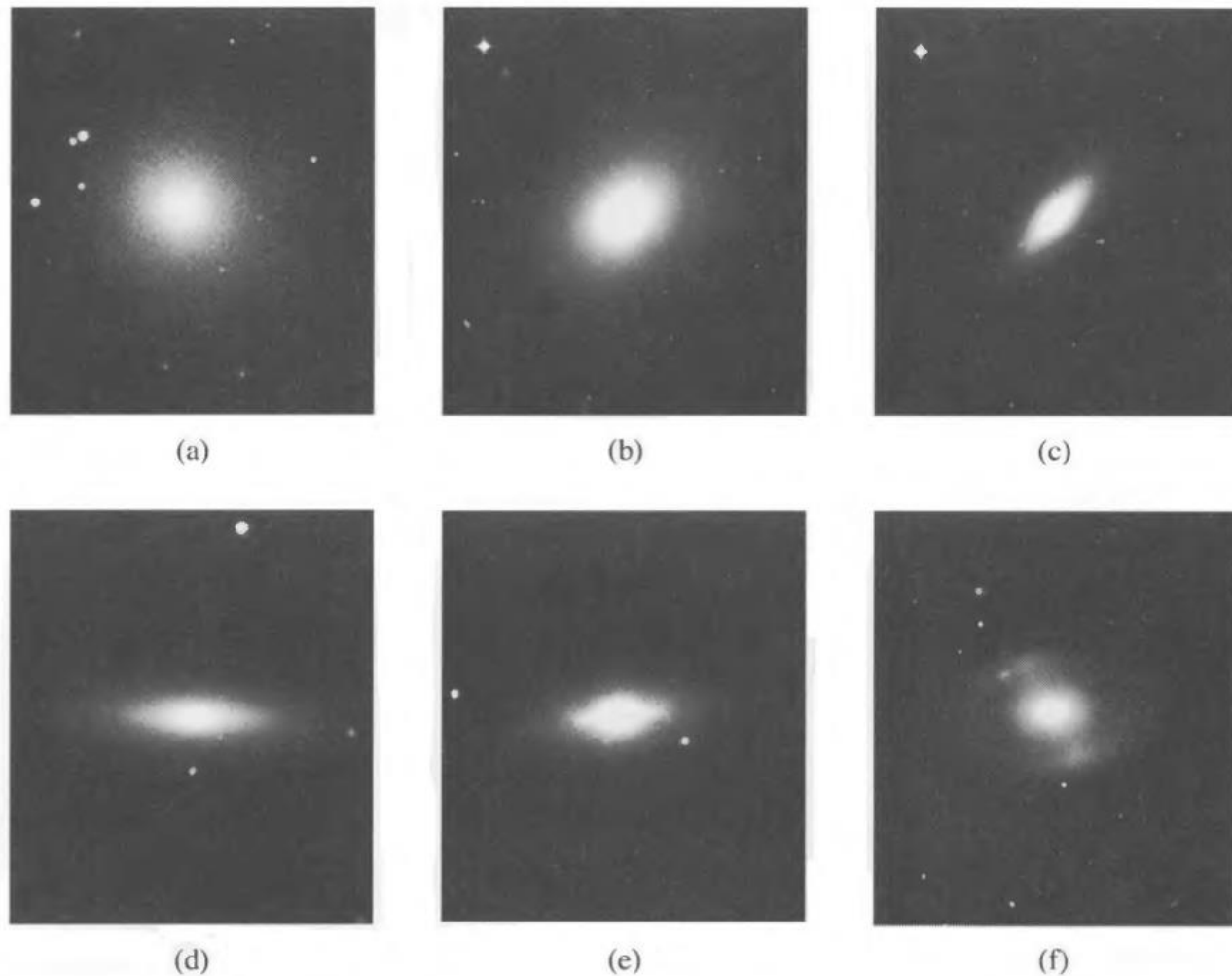


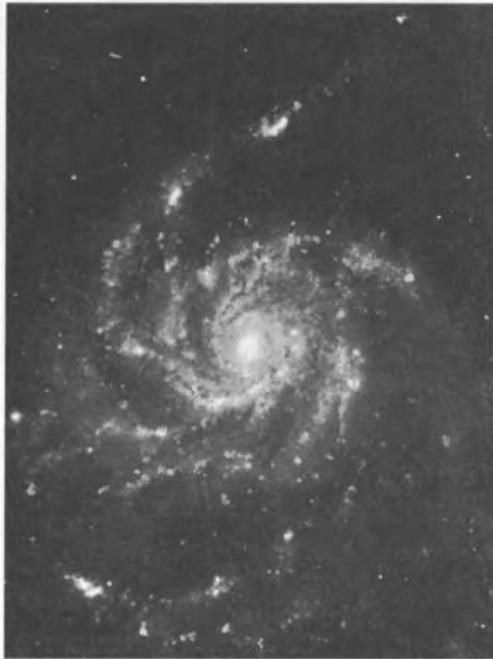
FIGURE 25.4 Typical early-type galaxies. (a) IC 4296 (E0), (b) NGC 4365 (E3), (c) NGC 4564 (E6), (d) NGC 4623 (E7), (e) NGC 4251 (S0), (f) NGC 4340 (RSB0). (Images from Sandage and Bedke, *The Carnegie Atlas of Galaxies*, Carnegie Institution of Washington, Washington, D.C., 1994.)



(a)



(b)

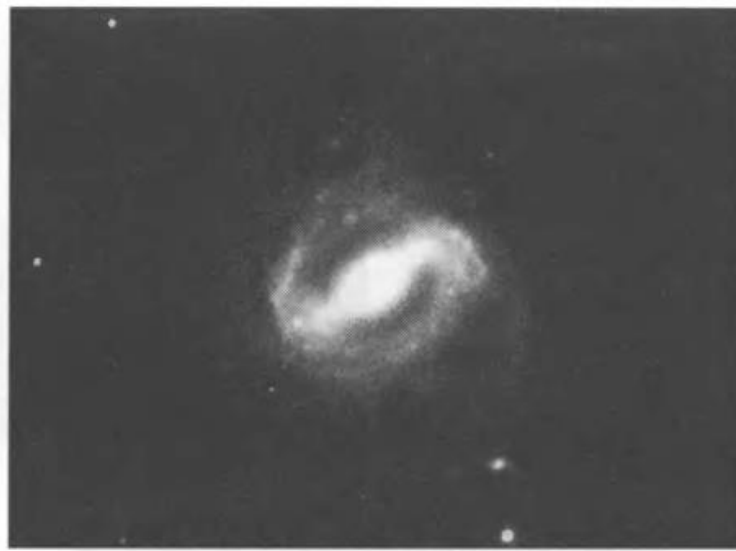


(c)



(d)

FIGURE 25.5 Typical normal spirals. (a) NGC 7096 (Sa(r)I), (b) M81/NGC 3031 (Sb(r)I-II), (c) M101/NGC 5457/Pinwheel (Sc(s)I), (d) M104/NGC 4594/Sombrero (Sa/Sb) seen nearly edge on. (Images from Sandage and Bedke, *The Carnegie Atlas of Galaxies*, Carnegie Institution of Washington, Washington, D.C., 1994.)



(a)

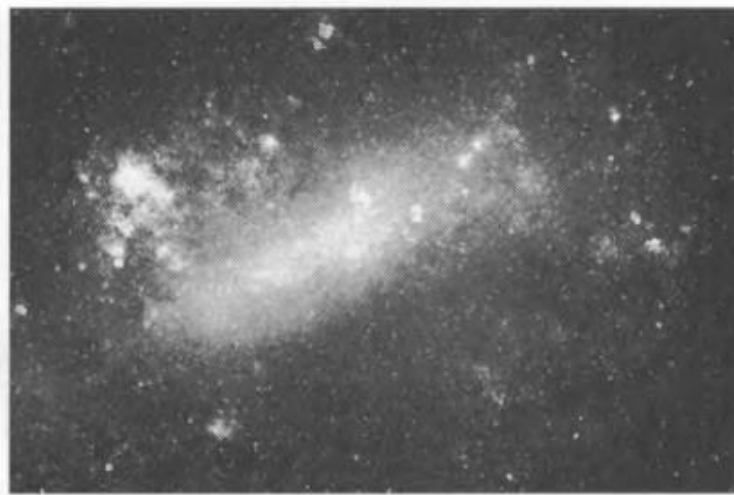


(b)



(c)

FIGURE 25.6 Typical barred spirals. (a) NGC 175 (SBab(s) I–II), (b) NGC 1300 (SBb(s) I), (c) NGC 2525 (SBc(s)II). (Images from Sandage and Bedke, *The Carnegie Atlas of Galaxies*, Carnegie Institution of Washington, Washington, D.C., 1994.)



(a)



(b)



(c)

FIGURE 25.7 Examples of irregular galaxies. (a) The Large Magellanic Cloud (Irr I/SBmIII), (b) the Small Magellanic Cloud (Irr I/ImIV–V), (c) M82/NGC 3034 (Irr II/Ir/Amorphous). (Images from Sandage and Bedke, *The Carnegie Atlas of Galaxies*, Carnegie Institution of Washington, Washington, D.C., 1994.)

TABLE 25.1 Characteristics of Early Spiral Galaxies.

	Sa	Sb	Sc
M_B	-17 to -23	-17 to -23	-16 to -22
M ($M_\odot$)	10^9 - 10^{12}	10^9 - 10^{12}	10^9 - 10^{12}
$\langle L_{\text{bulge}}/L_{\text{total}} \rangle_B$	0.3	0.13	0.05
Diameter (D_{25} , kpc)	5-100	5-100	5-100
$\langle M/L_B \rangle$ ($M_\odot/L_\odot$)	6.2 ± 0.6	4.5 ± 0.4	2.6 ± 0.2
$\langle V_{\text{max}} \rangle$ (km s^{-1})	299	222	175
V_{max} range (km s^{-1})	163-367	144-330	99-304
pitch angle	$\sim 6^\circ$	$\sim 12^\circ$	$\sim 18^\circ$
$\langle B - V \rangle$	0.75	0.64	0.52
$\langle M_{\text{gas}}/M_{\text{total}} \rangle$	0.04	0.08	0.16
$\langle M_{\text{H}_2}/M_{\text{H I}} \rangle$	2.2 ± 0.6 (Sab)	1.8 ± 0.3	0.73 ± 0.13
$\langle S_N \rangle$	1.2 ± 0.2	1.2 ± 0.2	0.5 ± 0.2

TABLE 25.2 Characteristics of Late Spiral and Irregular Galaxies.

	Sd/Sm	Im/Ir
M_B	-15 to -20	-13 to -18
M ($M_\odot$)	10^8 - 10^{10}	10^8 - 10^{10}
Diameter (D_{25} , kpc)	0.5-50	0.5-50
$\langle M/L_B \rangle$ ($M_\odot/L_\odot$)	~ 1	~ 1
V_{max} range (km s^{-1})	80-120	50-70
$\langle B - V \rangle$	0.47	0.37
$\langle M_{\text{gas}}/M_{\text{total}} \rangle$	0.25 (Scd)	0.5-0.9
$\langle M_{\text{H}_2}/M_{\text{H I}} \rangle$	0.03-0.3	~ 0
$\langle S_N \rangle$	0.5 ± 0.2	0.5 ± 0.2

Surface Brightness Profiles

As a result of the decrease in the **density** of **stars**, the surface brightnesses of galaxies falls with increasing radius.

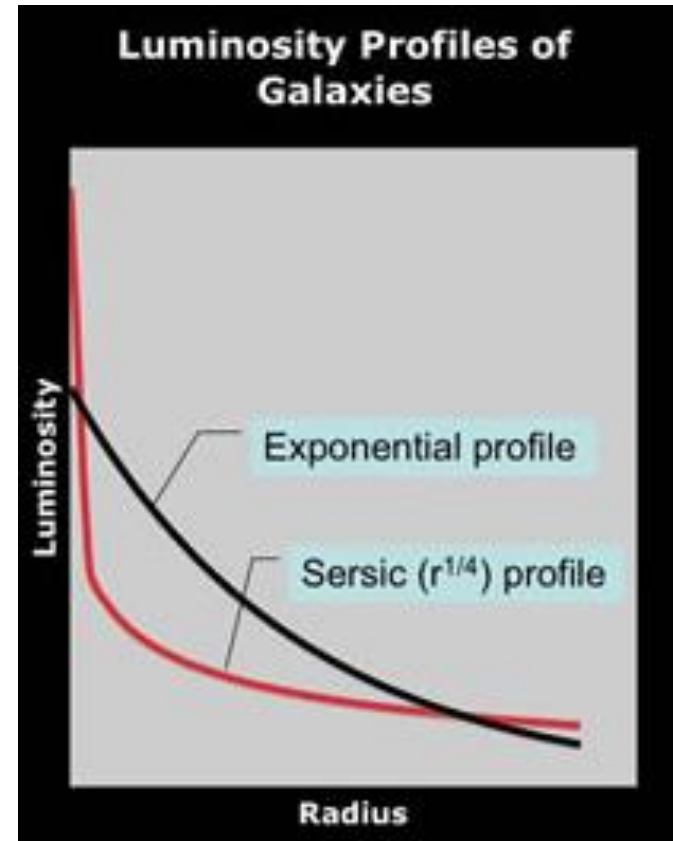
The way in which the brightness falls is described by the surface brightness profile.

The two commonly observed **surface brightness profiles** for galaxies – the **exponential** and **Sersic profiles** – are shown in the image to the right.

Exponential galaxies have surface brightness profiles described by an equation of the form:

$$I(r) = I_0 e^{-\beta \frac{r}{r_e}}$$

Where $I(r)$ is the surface brightness at radius r , I_0 is the central brightness, and r_e is the scale length. β is chosen such that one-half of the total light of the system would be emitted interior to r_e . *The disks of spiral galaxies generally exhibit exponential surface brightness profiles.*

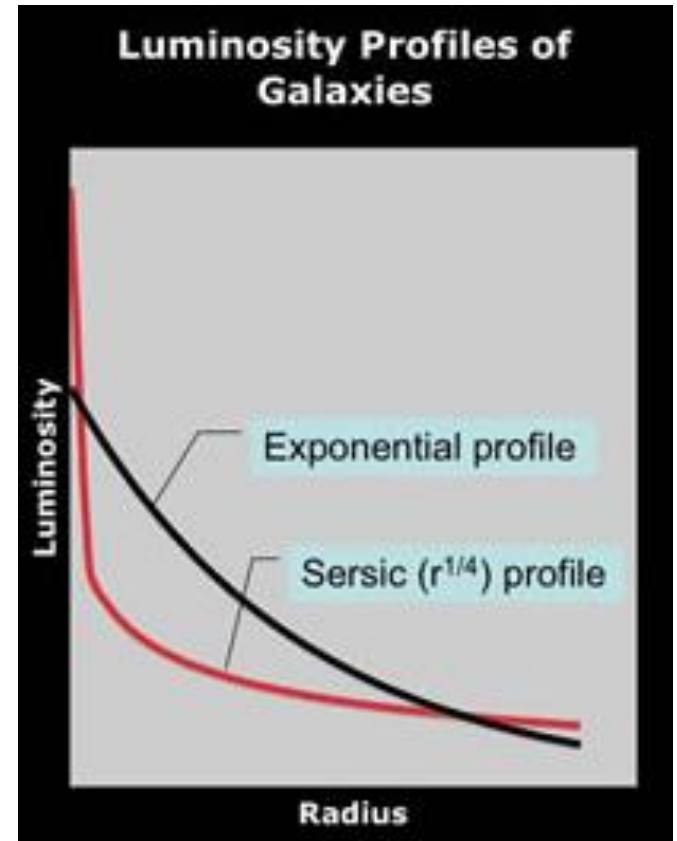


Surface Brightness Profiles

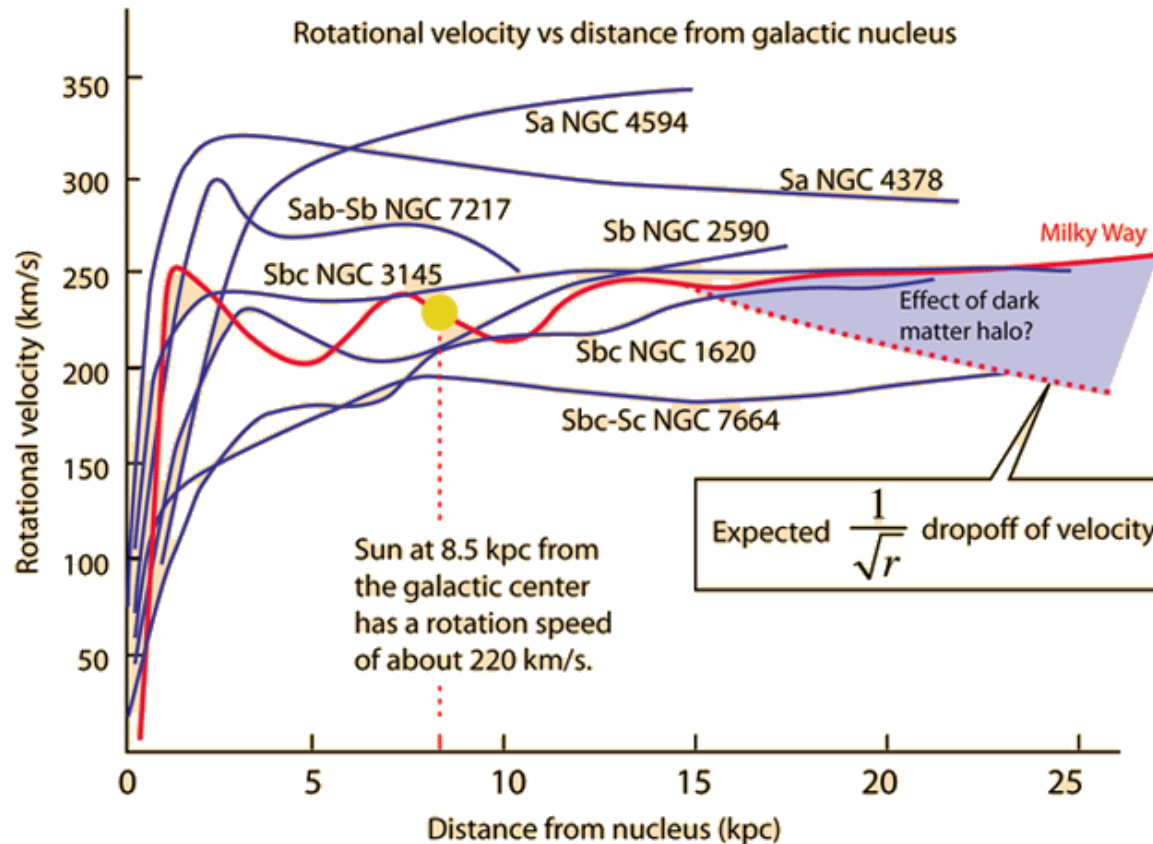
Galaxies with **Sersic profiles** possess surface brightness profiles described by an equation of the form:

$$I(r) = I_0 e^{-\beta \left(\frac{r}{r_e}\right)^{1/n}}$$

Where n is a number between approximately 2 and 6, and all other parameters are as above. **Elliptical galaxies**, and the bulges of spiral galaxies, generally **exhibit Sersic surface brightness profiles with a typical n of 4**. This is the famous de Vaucouleurs $r^{1/4}$ profile.



Rotation Curves of Galaxies



The analysis of rotation curves provides a direct means of determining the total distribution of all matter (dark and luminous) in a galaxy.

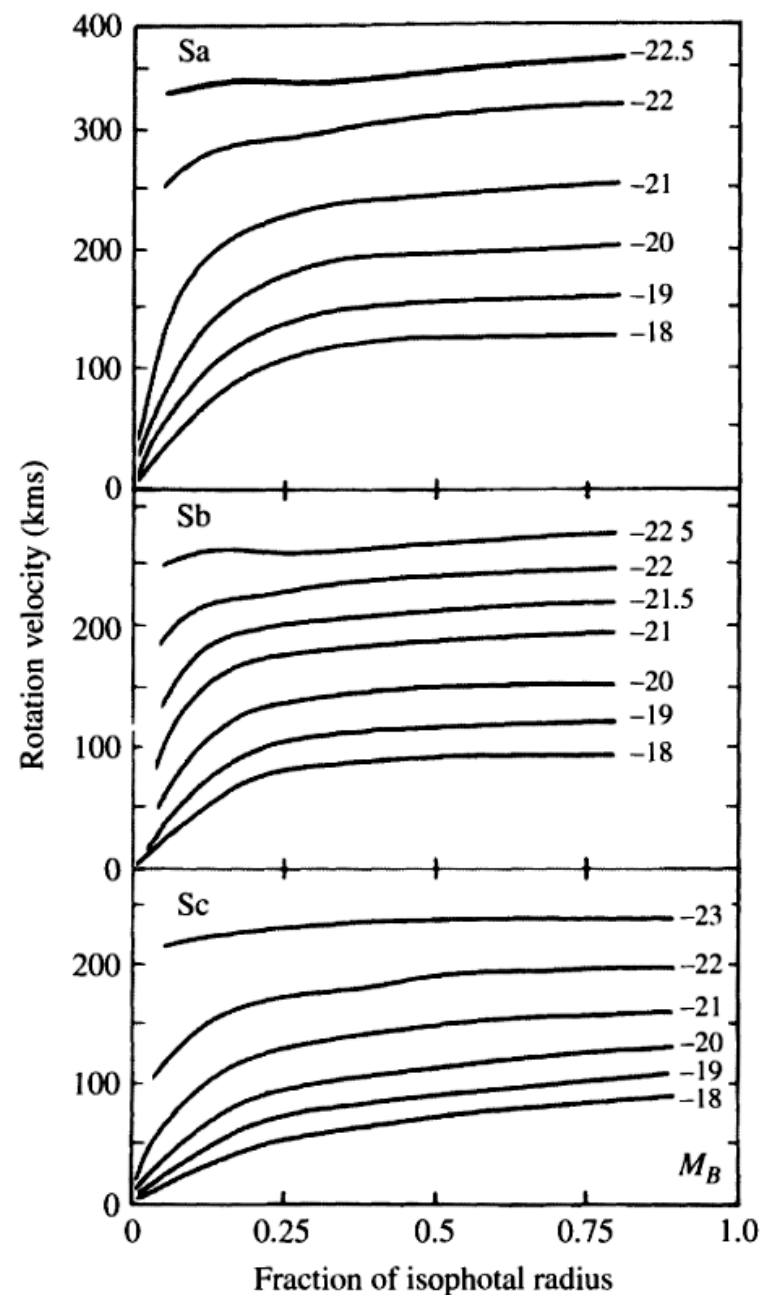


FIGURE 25.8 Variations in the average rotation curves of Sa, Sb, and Sc galaxies for various values of absolute magnitude in the B band. (Figure adapted from Rubin et al., *Ap. J.*, 289, 81, 1985.)

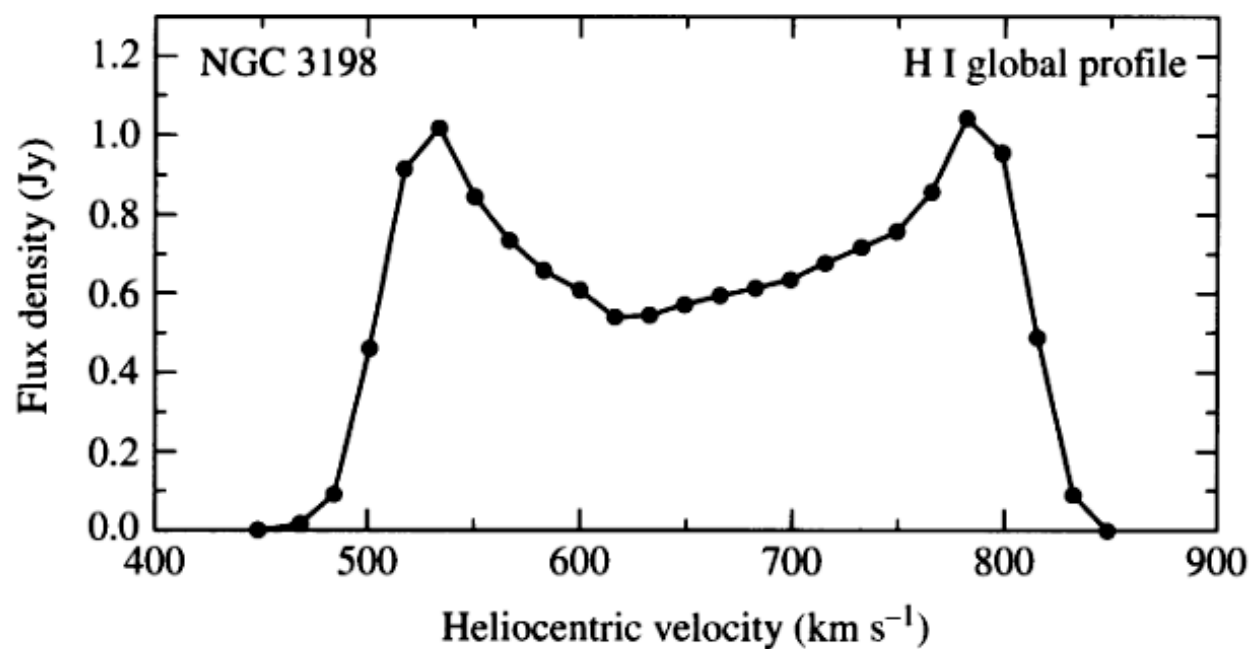


FIGURE 25.9 The global H I profile for NGC 3198. (Data from Begeman, *Astron. Astrophys.*, 223, 81, 1991.)

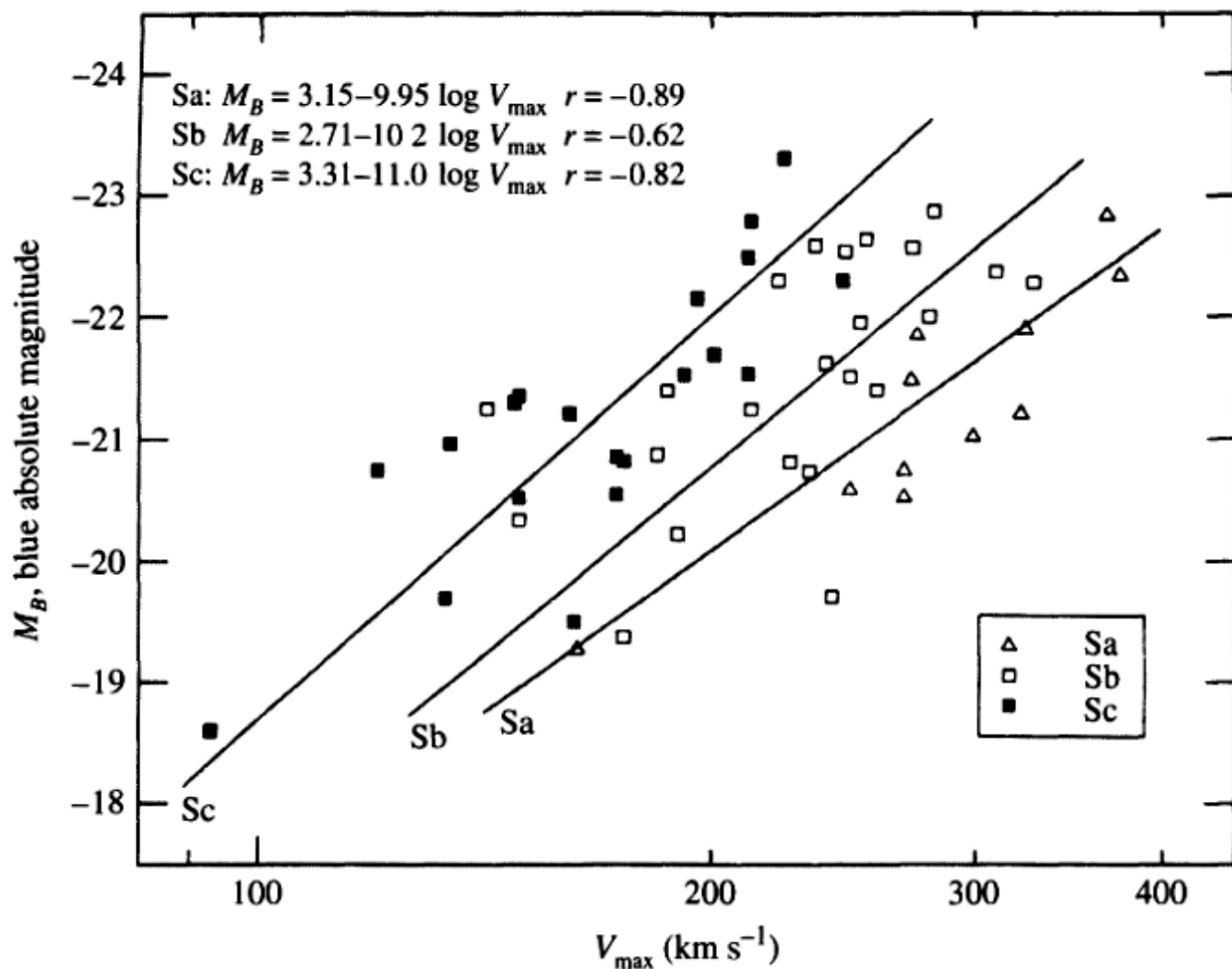


FIGURE 25.10 The Tully–Fisher relation for early spiral galaxies. (Figure adapted from Rubin et al., *Ap. J.*, 289, 81, 1985.)

Tully – Fisher Relation

An empirical **relation exists between the luminosity** of a spiral galaxy and its **maximum rotation velocity**.

$$L_{spiral} \propto v_{max}^4$$

Where v_{max} is the maximum rotational velocity

The T-F relation was first determined in 1977 by Tully and Fisher from measured profiles of the 21 cm emission lines of neutral hydrogen in a sample of spiral galaxies

Tully – Fisher Relation

Derivation of : $L_{spiral} \propto v_{max}^4$

For the flat rotation curves of spiral galaxies:

$$\frac{mv^2}{R} = \frac{GMm}{R^2} \Rightarrow M = \frac{v^2 R}{G}$$

Assumption 1: Mass-to-Light ratio of Spiral galaxies is constant

$$\frac{M}{L} = c_1 \Rightarrow L = \frac{M}{c_1} = \frac{1}{c_1} \frac{v_{max}^2 R}{G} \quad (1)$$

Assumption 2: Assume that the surface brightness at the centers of all spirals galaxies is the same

$$I = \frac{L}{Area} \propto \frac{L}{R^2} \Rightarrow R^2 = \frac{L}{c_2} \Rightarrow R = \frac{L^{1/2}}{c_2^{1/2}} \quad (2)$$

$$(1) \wedge (2) \Rightarrow L = \frac{1}{c_1} \frac{v_{max}^2}{G} \frac{L^{1/2}}{c_2^{1/2}} \Rightarrow L^{1/2} = \frac{v_{max}^2}{c_3} \Rightarrow L = c_4 v_{max}^4$$

Tully – Fisher Relation

Observed correlations between absolute magnitudes and $v_{\max}$ of spiral galaxies:

$$M_B = -9.95 \log v_{\max} + 3.15 \text{ for } S_a$$

$$M_B = -10.2 \log v_{\max} + 2.7 \text{ for } S_b$$

$$M_B = -11 \log v_{\max} + 3.31 \text{ for } S_c$$

$$M - M_{\odot} = 2.5 \log \frac{L_{\odot}}{L} \Rightarrow M = M_{\odot} - 2.5 \log L + 2.5 \log L_{\odot} \Rightarrow$$

$$M = -2.5 \log c v_{\max}^4 + M_{\odot} + 2.5 \log L_{\odot} \Rightarrow$$

$$M = -10 \log v_{\max} + \text{constant}$$

Estimate the total mass near the center of a Galaxy

Virial theorem: $-2\langle K \rangle = \langle U \rangle$

$$K = \sum_1^N \frac{1}{2} m_i v_i^2$$

$$U = \sum_1^N u_i$$

$$-2 \frac{1}{2} m \sum_1^N v_i^2 = U \Rightarrow -m \langle v^2 \rangle = \frac{U}{N} \quad (1)$$

$$\langle v^2 \rangle = \langle v_r^2 \rangle + \langle v_\theta^2 \rangle + \langle v_\phi^2 \rangle = 3 \langle v_r^2 \rangle \quad (2)$$

$$(1) \wedge (2) \Rightarrow -m 3 \langle v_r^2 \rangle = \frac{U}{N} \quad (3)$$

Velocity Dispersion: $\sigma^2 = \frac{1}{N} \sum_1^N (v_i - \langle v \rangle)^2 = \frac{1}{N} \sum_1^N (v_i)^2 = \langle v^2 \rangle \Rightarrow$
 $\sigma_r^2 = \langle v_r^2 \rangle \quad (4)$

$$(3) \wedge (4) \Rightarrow -3m\sigma_r^2 = \frac{U}{N} \quad (5)$$

Estimate the mass near the center of a Galaxy

Derivation of total potential energy of N stars of mass m that are uniformly distributed within a sphere of radius R

$$dU = -\frac{GM(< r)dm}{r}$$

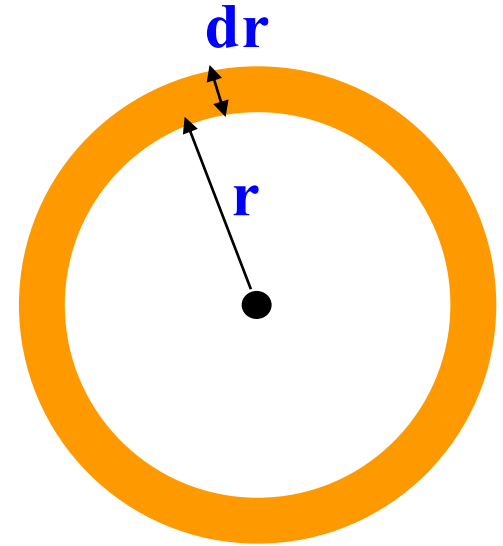
$$M(< r) = \rho \frac{4}{3} \pi r^3$$

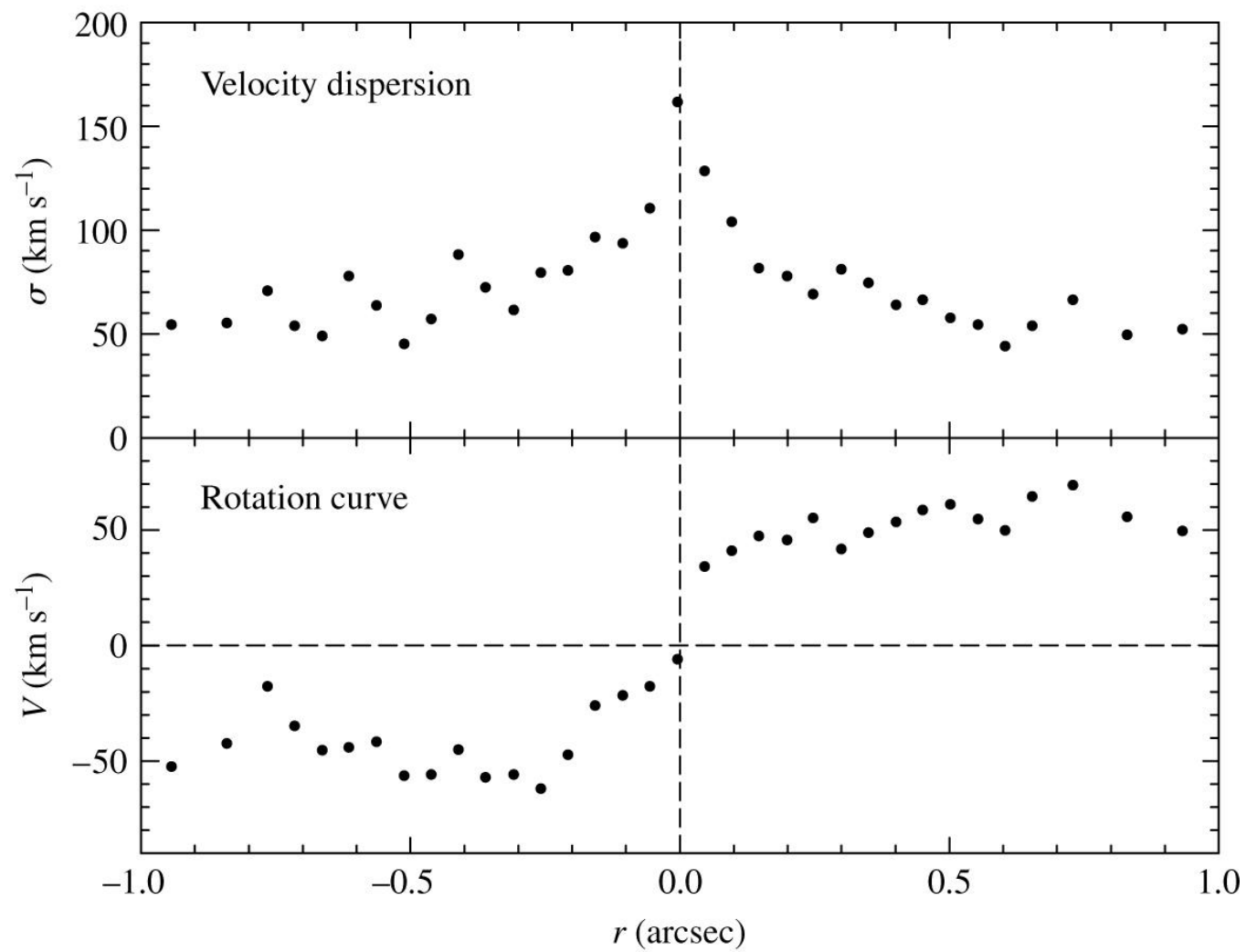
$$dm = 4\pi r^2 dr \rho$$

$$U = \int_0^R dU(r) = - \int_0^R G \frac{\rho \frac{4}{3} \pi r^3 4\pi r^2 dr \rho}{r} =$$
$$= G \frac{(4\pi)^2}{3} \rho^2 \int_0^R r^4 dr \Rightarrow U = -\frac{3}{5} \frac{GM^2}{R} \quad (6)$$

$$(5) \wedge (6) \Rightarrow -3m\sigma_r^2 = -\frac{3}{5} \frac{GM^2}{RN} \Rightarrow$$

$$M_{\text{Virial}} = \frac{5R\sigma_r^2}{G}$$





Specific Frequency of Globular Clusters: S_N

$$S_N = N_t \frac{L_{15}}{L_V} = N_t 10^{0.4(M_V + 15)}$$

Where:

N_t is the number of globular clusters in the galaxy

L_{15} is the luminosity that corresponds to a V magnitude of $M_V = -15$

L_V is the V band luminosity of the galaxy

M_V is the absolute V band magnitude of the galaxy

Specific Frequency of Globular Clusters: S_N

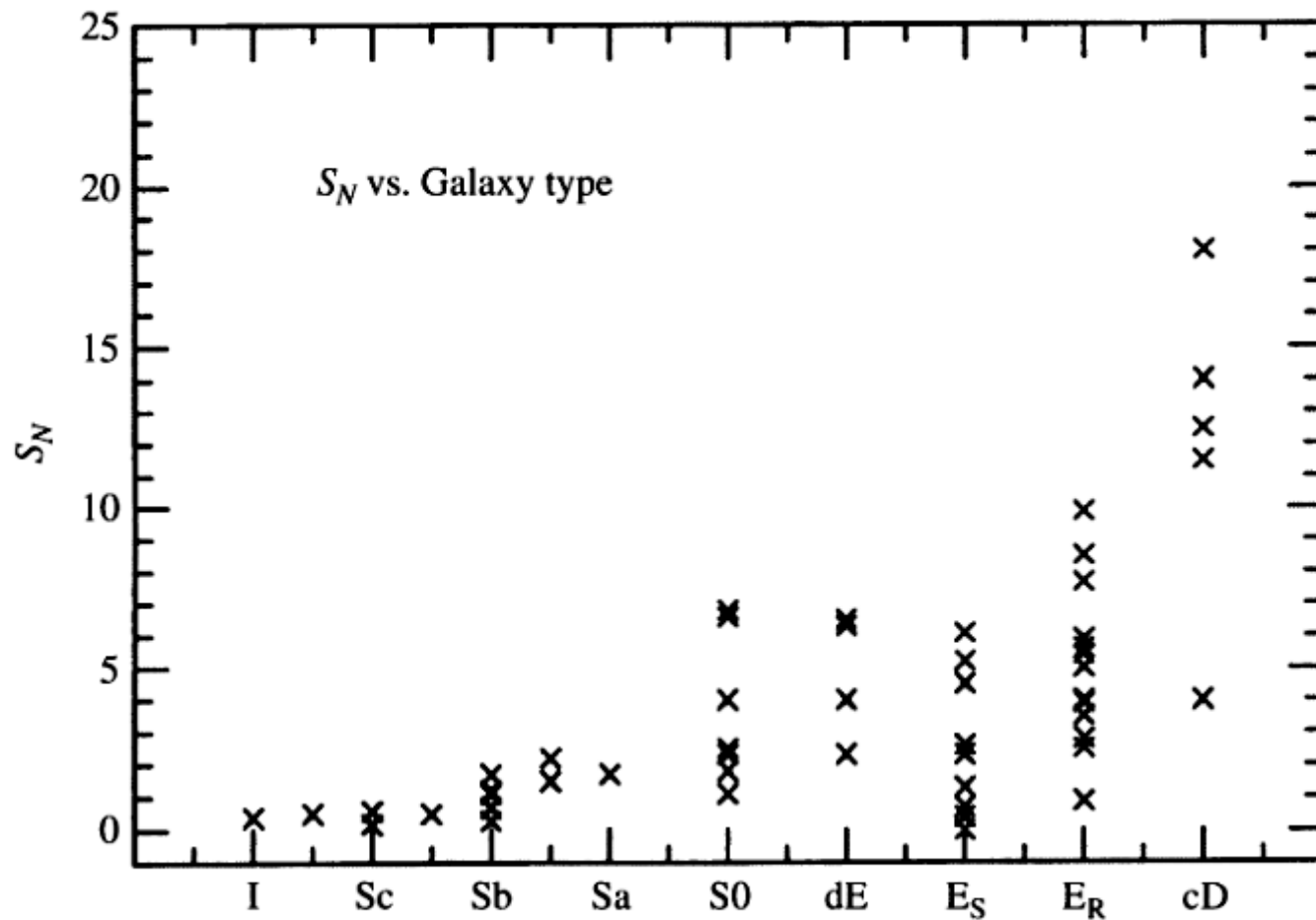
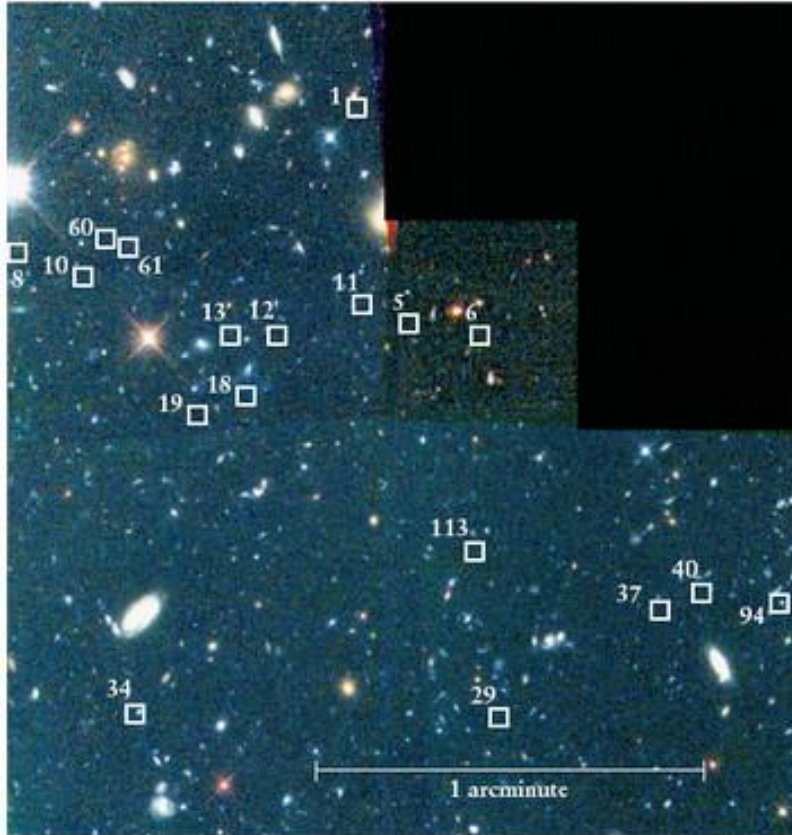


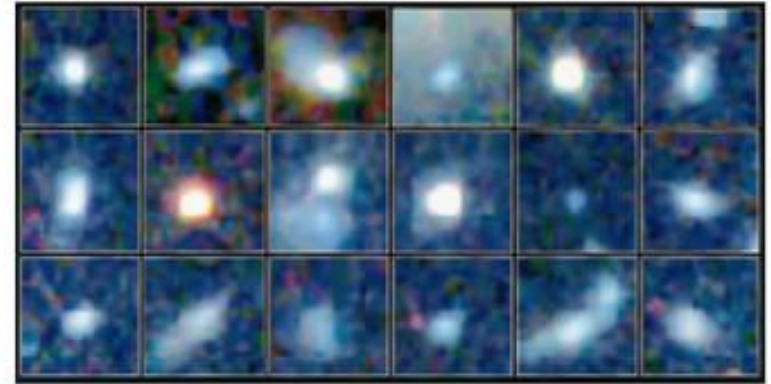
FIGURE 25.16 The association of the specific frequency of globular clusters with Hubble type. cD galaxies are the largest elliptical galaxies in the universe; dE's are dwarf ellipticals. (Figure adapted from Harris, *Annu. Rev. Astron. Astrophys.*, 29, 543, 1991. Reproduced by permission from the *Annual Review of Astronomy and Astrophysics*, Volume 29, ©1991 by Annual Reviews Inc.)

Formation of the First Galaxies



(a) A portion of the constellation Hercules

These very deep observations with HST show us how galaxies looked like in the distant past some ~ 11 billion years ago.



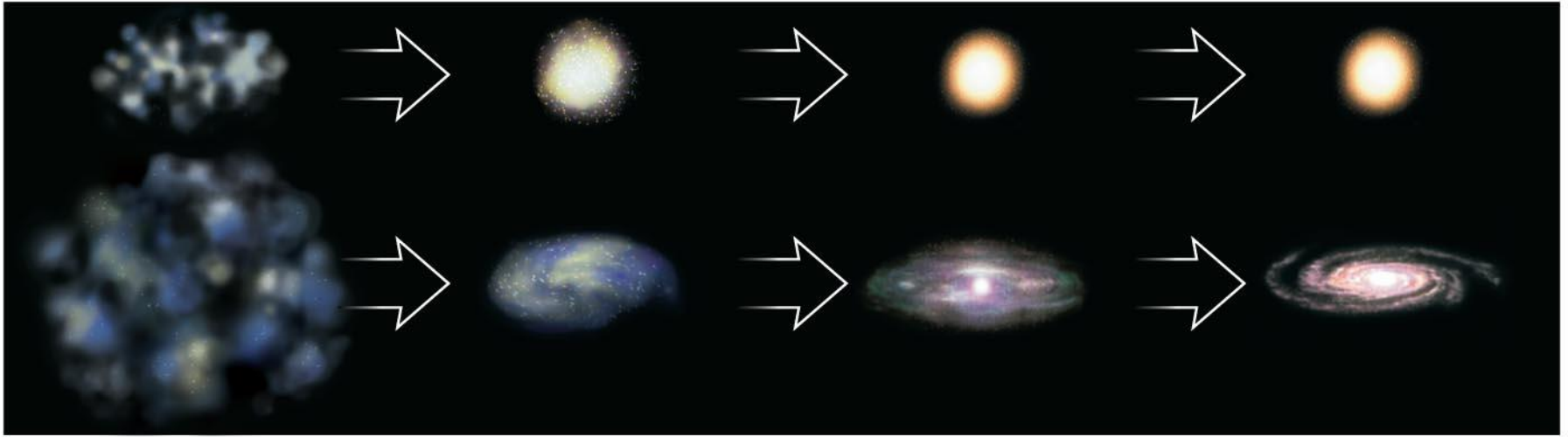
(b) Closeup images of the numbered objects in (a)

Recent observations of the very first galaxies in the Universe suggest that **galaxies formed from the merger of smaller objects**. This is often referred to as building galaxies from the "Bottom Up". The building blocks of galaxies are about a few 1000 ly across.

The **very first galaxies appear very blue** indicating significant star formation at this time.

Why do galaxies differ?
What are starbursts?

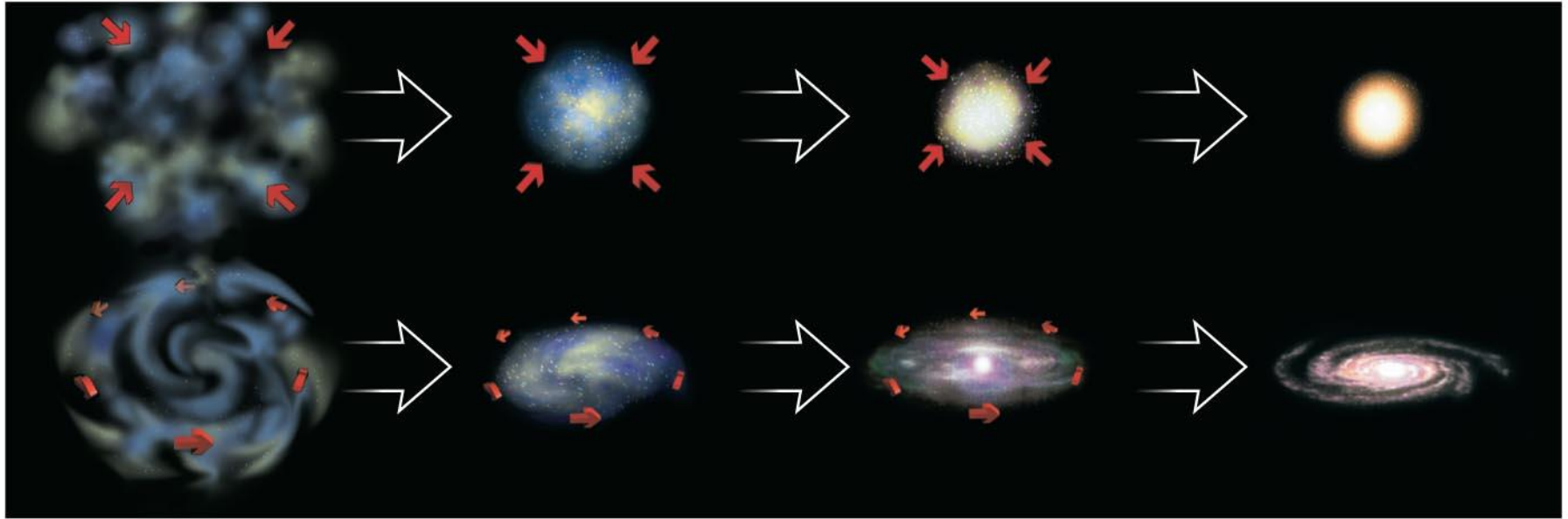
Why do galaxies differ?



b The gas density of a galaxy's protogalactic clouds may determine whether it ends up spiral or elliptical.

1. Birth Conditions (rotation, density)
2. Mergers

Conditions in Protogalactic Cloud?

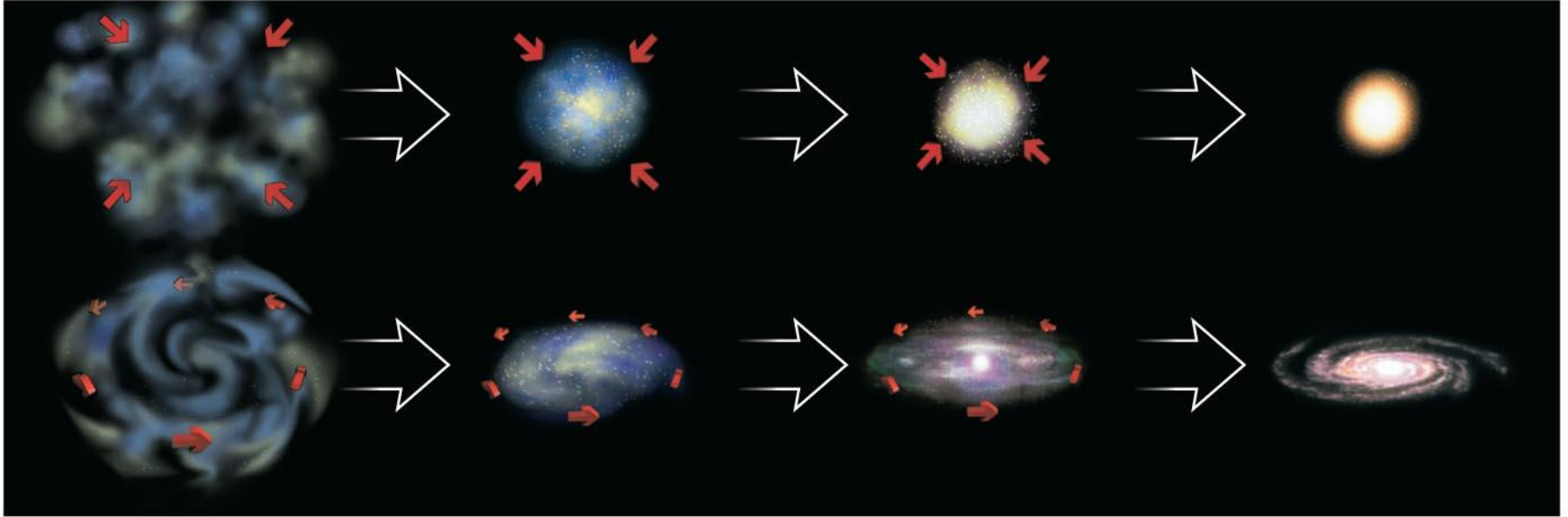


a The angular momentum of a galaxy's protogalactic-cloud system may determine whether it ends up spiral or elliptical.

Interactive Figure 

- *Spin*: The initial angular momentum of the protogalactic cloud could determine the size of the resulting disk. Protogalactic clouds with large spin are likely to form a disk and the resulting galaxy will be spiral.

Conditions in Protogalactic Cloud?

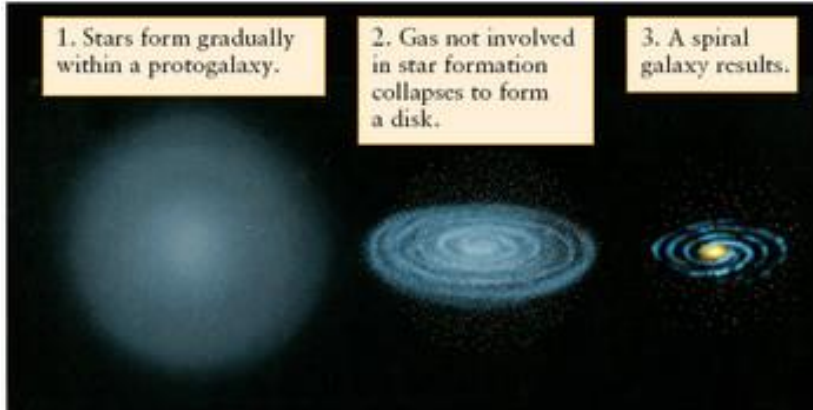


a The angular momentum of a galaxy's protogalactic-cloud system may determine whether it ends up spiral or elliptical.

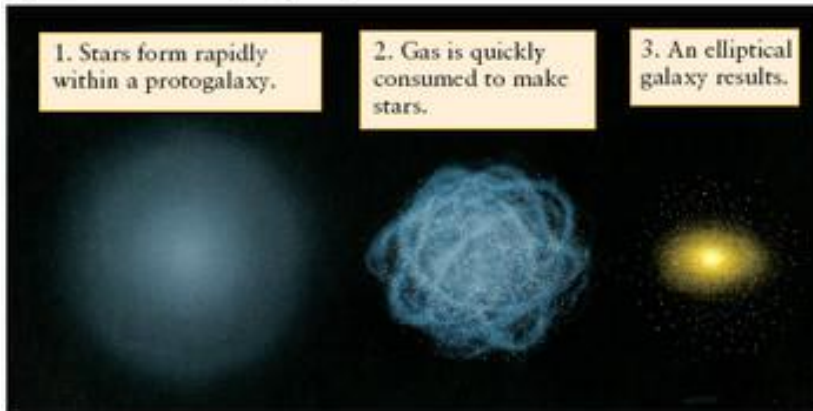
Interactive Figure 

- **Density:** Elliptical galaxies could come from dense protogalactic clouds that were able to radiate energy and cool, forming stars before gas settled into a disk.

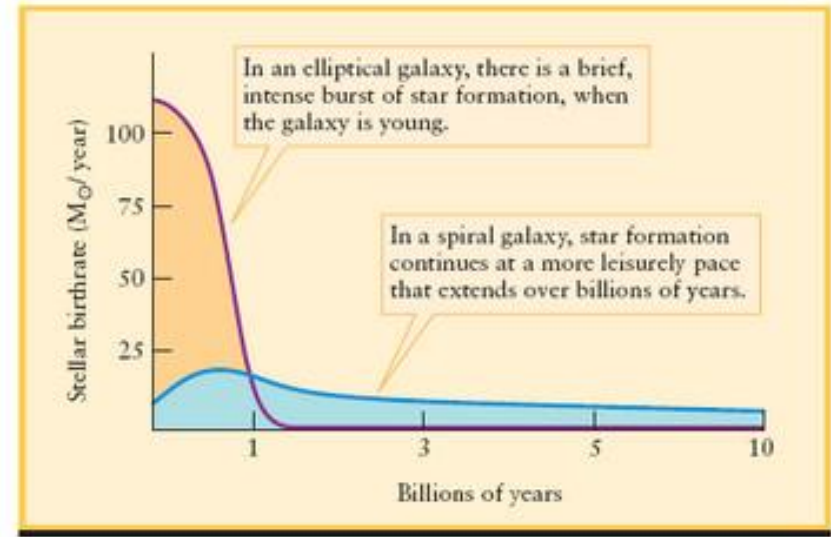
Spiral versus Elliptical Galaxies



(a) Formation of a spiral galaxy



(b) Formation of an elliptical galaxy

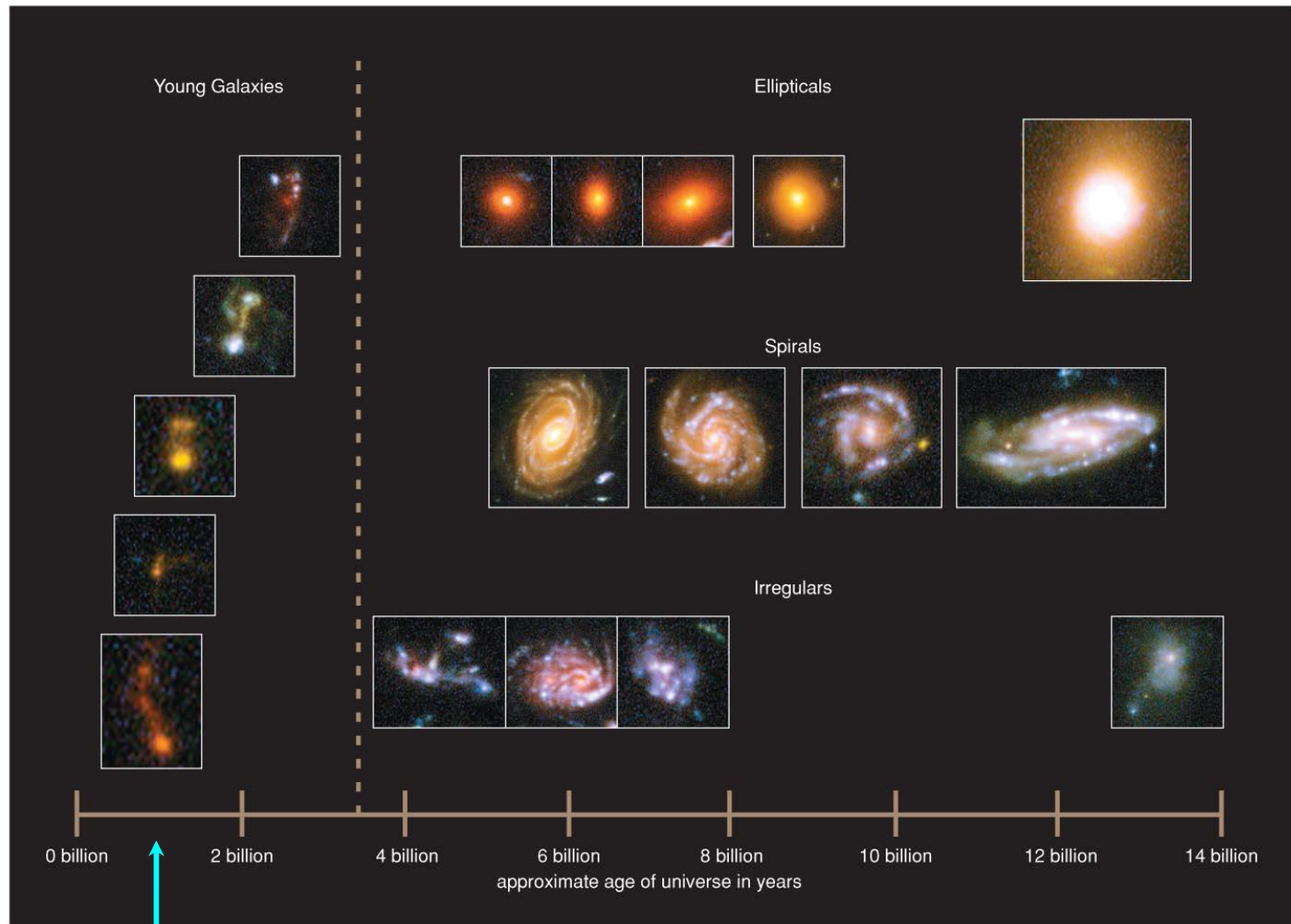


(c) The stellar birthrate in galaxies

One idea to explain why some galaxies become spirals and other ellipticals is that if star formation is large during the formation of the protogalaxy all the gas will be consumed quickly to form stars and no disk is formed resulting in an elliptical galaxy. Conversely, if star formation is weak within a protogalaxy the gas will have time to settle and form a disk resulting in a spiral galaxy.



- We must also consider the effects of collisions.



- Collisions were much more likely early in time because galaxies were closer together.

A composite image of **M51**, also known as the **Whirlpool Galaxy**.

X-ray data from the Chandra X-ray Observatory reveals point-like sources (**purple**) that are black holes and neutron binary systems. Chandra also detects a diffuse glow of hot gas that permeates the space between the stars.

Optical data from the Hubble Space Telescope (**green**) and **infrared emission** from the Spitzer Space Telescope (**red**) both highlight long lanes in the spiral arms that consist of stars and gas laced with dust.

A view of M51 with the GALEX telescope shows hot, young stars that produce lots of **ultraviolet** energy (**blue**).



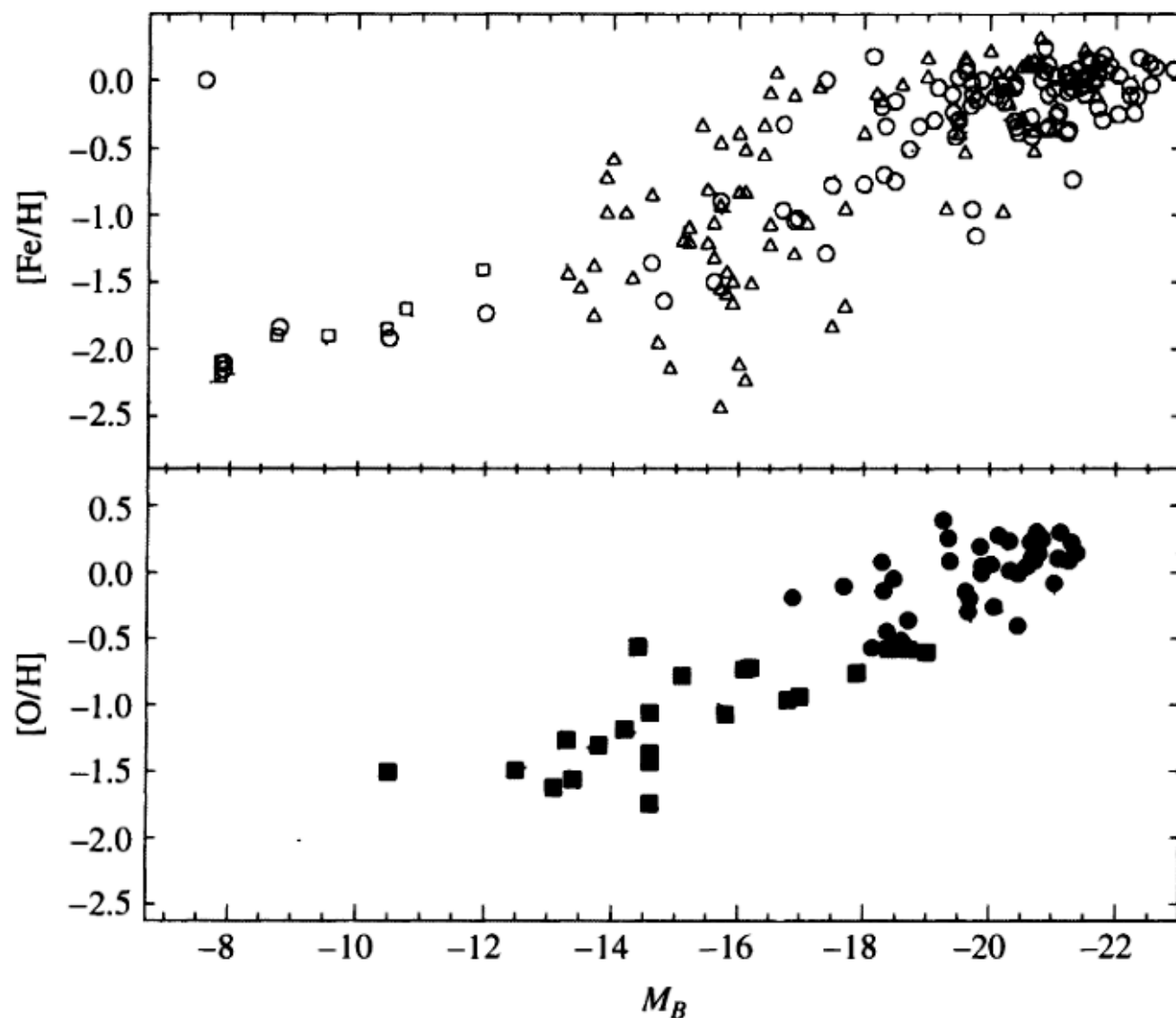
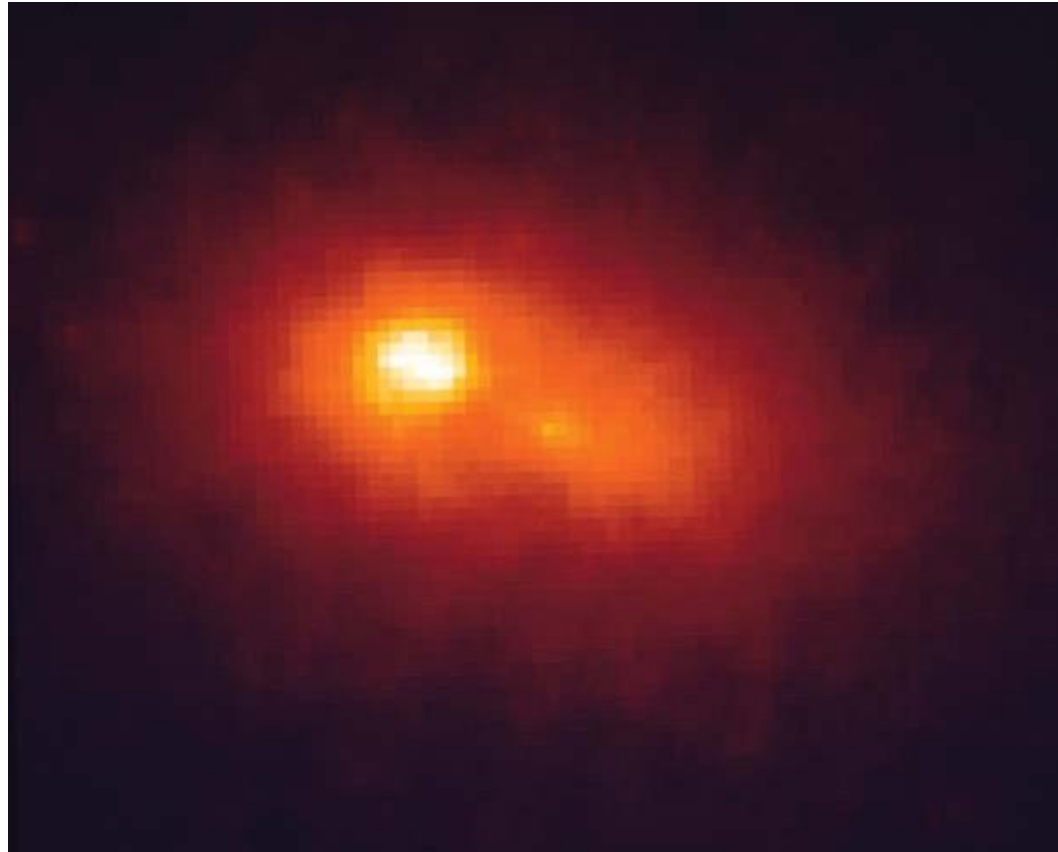


FIGURE 25.12 Metallicity is observed to increase linearly with absolute magnitude. The symbols in the upper panel correspond to dE's (dwarf ellipticals, open squares) and ellipticals (open circles and open triangles). In the lower panel, spirals are represented by filled circles and irregulars are indicated by filled squares. (Figure adapted from Zaritsky, Kennicutt, and Huchra, *Ap. J.*, 420, 87, 1994.)

A Black Hole Census



Hubble image of the central region of the Andromeda galaxy (M31) in visible light. The stars form an eccentric disk around the SMBH ($M_{\text{BH}} \sim 140 \times 10^6 M_{\odot}$). The black hole is located closer to the fainter of the two peaks in the light distribution.

Black Hole Mass and Galaxy Bulge Mass

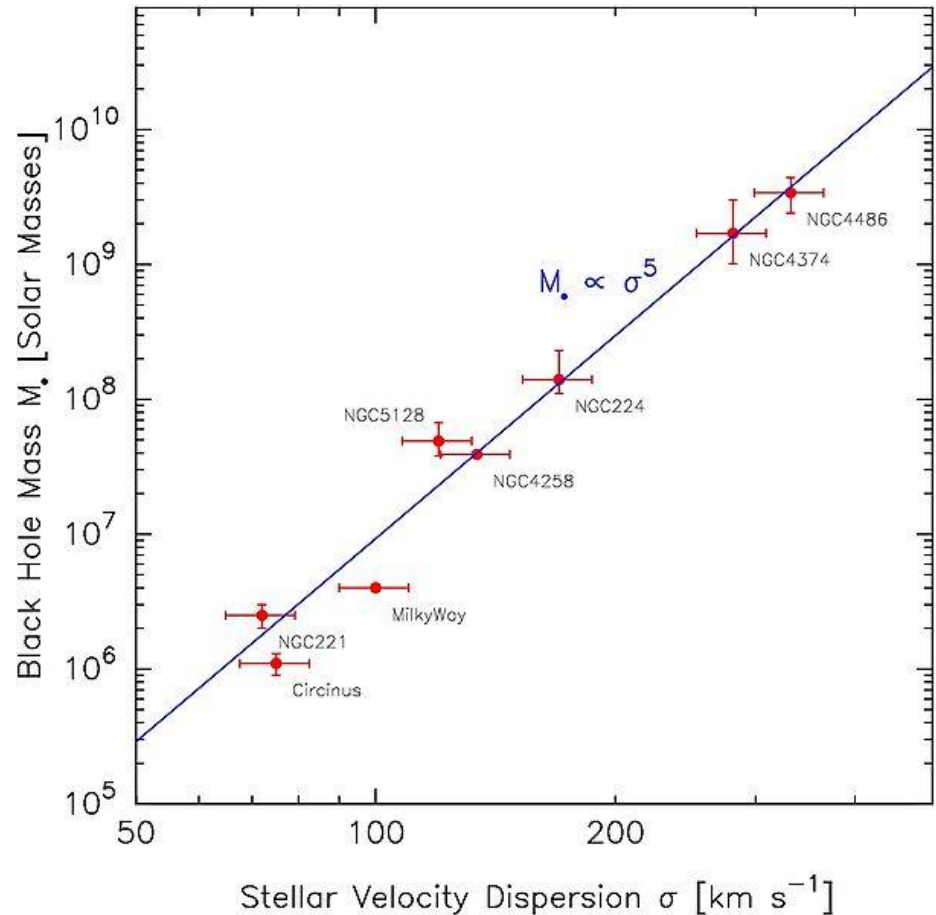
Observations in the 1990s had demonstrated a possible empirical **relationship between galaxy luminosity and black hole mass** but this relationship had large scatter (*Magorrian Relation*).

The mean ratio of black hole mass to bulge mass is believed to be approximately 0.1% , i.e., a bulge of one billion solar masses contains a black hole of approximately one million solar masses.

Black Hole Mass – Velocity Dispersion Relation

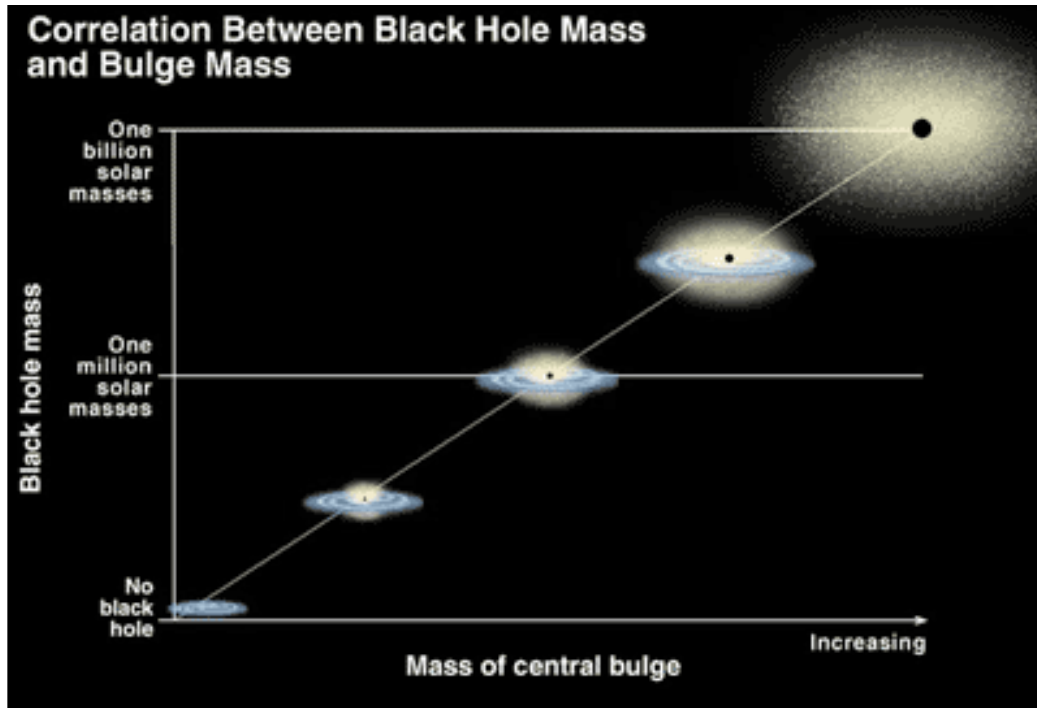
The $M_{\text{BH}}\text{-}\sigma$ relation is an empirical correlation between the stellar velocity dispersion σ of a galaxy bulge and the mass M of the supermassive black hole at the galaxy's center.

$$\sigma = \frac{1}{\sqrt{N}} \left[\sum_1^N (u - \langle u \rangle)^2 \right]^{1/2}$$

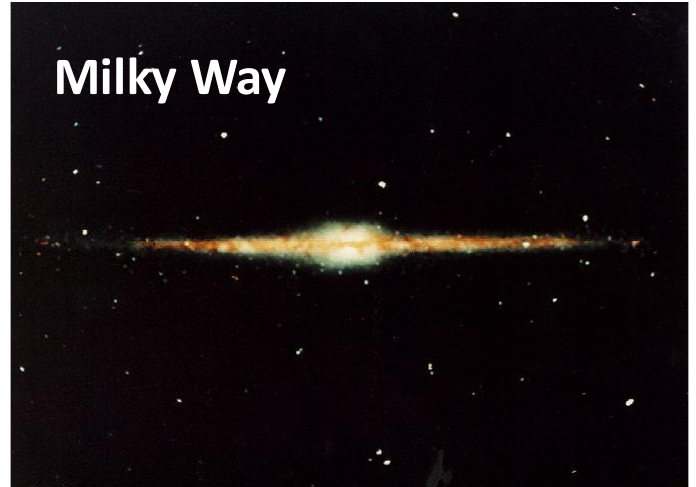


$$M_{\text{BH}} \propto \sigma^\alpha, \alpha \approx 5$$

Black Hole Mass – Velocity Dispersion Relation



Milky Way



Sombrero Galaxy

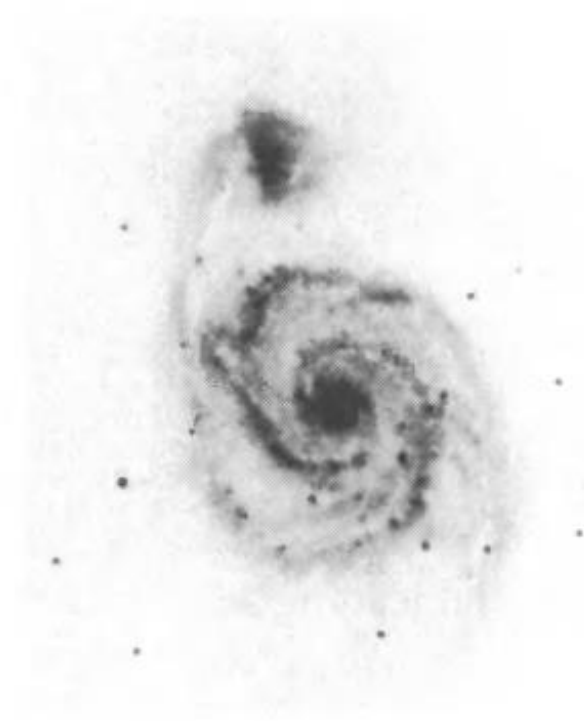




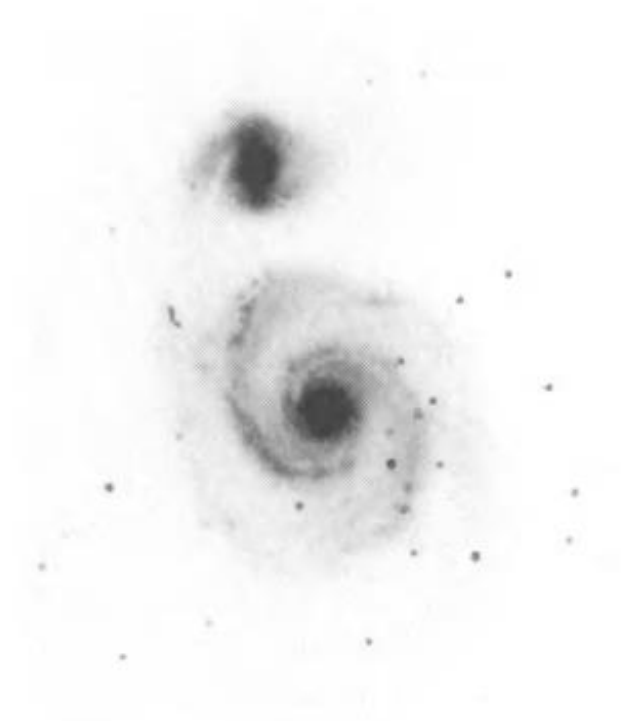
FIGURE 25.17 The Whirlpool galaxy, M51 (NGC 5194), is an Sbc(s)I–II grand-design spiral located in the constellation Canes Venatici (the Hunting Dogs of Boötes, just below Ursa Major). Also visible is its companion NGC 5195, situated near the end of one of the spiral arms. (Image from Sandage and Bedke, *The Carnegie Atlas of Galaxies*, Carnegie Institution of Washington, Washington, D.C., 1994.)



FIGURE 25.18 The Sb galaxy NGC 2841 is an example of a flocculent spiral. (Image from Sandage and Bedke, *The Carnegie Atlas of Galaxies*, Carnegie Institution of Washington, Washington, D.C., 1994.)



(a)



(b)

FIGURE 25.19 M51 as seen in (a) blue light and (b) red light. (Figures from Elmegreen, *Ap. J. Suppl.*, 47, 229, 1981.)

Spiral Structure

Spiral galaxies exhibit a variety of spiral structure:

- Number of arm varies
- How tightly wound the arms are varies
- Smoothness of stellar distribution varies

Spiral galaxies with very symmetric and well-defined spiral arms are called **grand-design spirals**

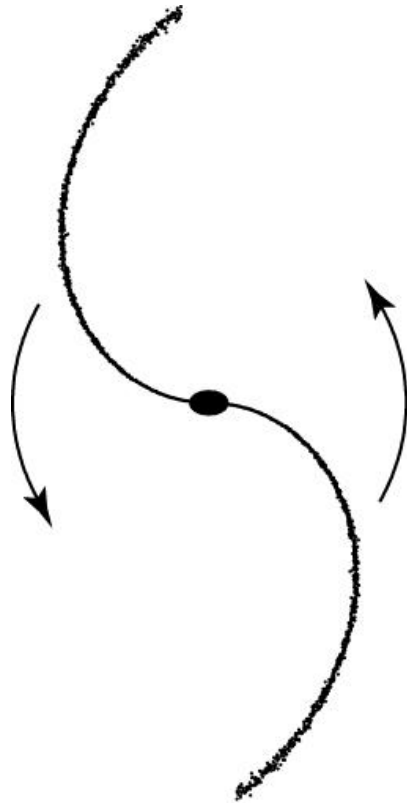
Spiral galaxies that do not possess well defined spiral arms are called **flocculent spirals**

10% Spirals are Grand Design

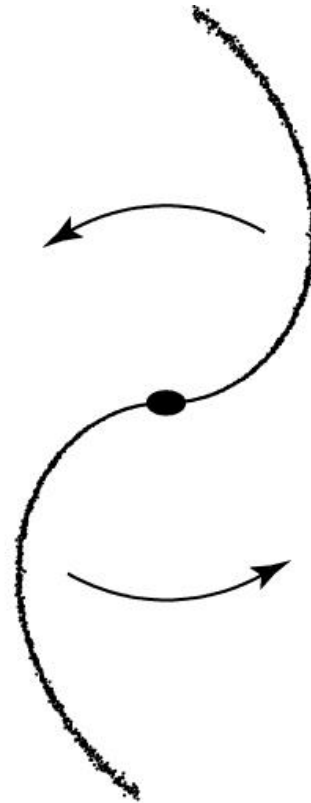
60% have multiple arms

30% are flocculent

Direction of Rotation of Spiral Galaxies



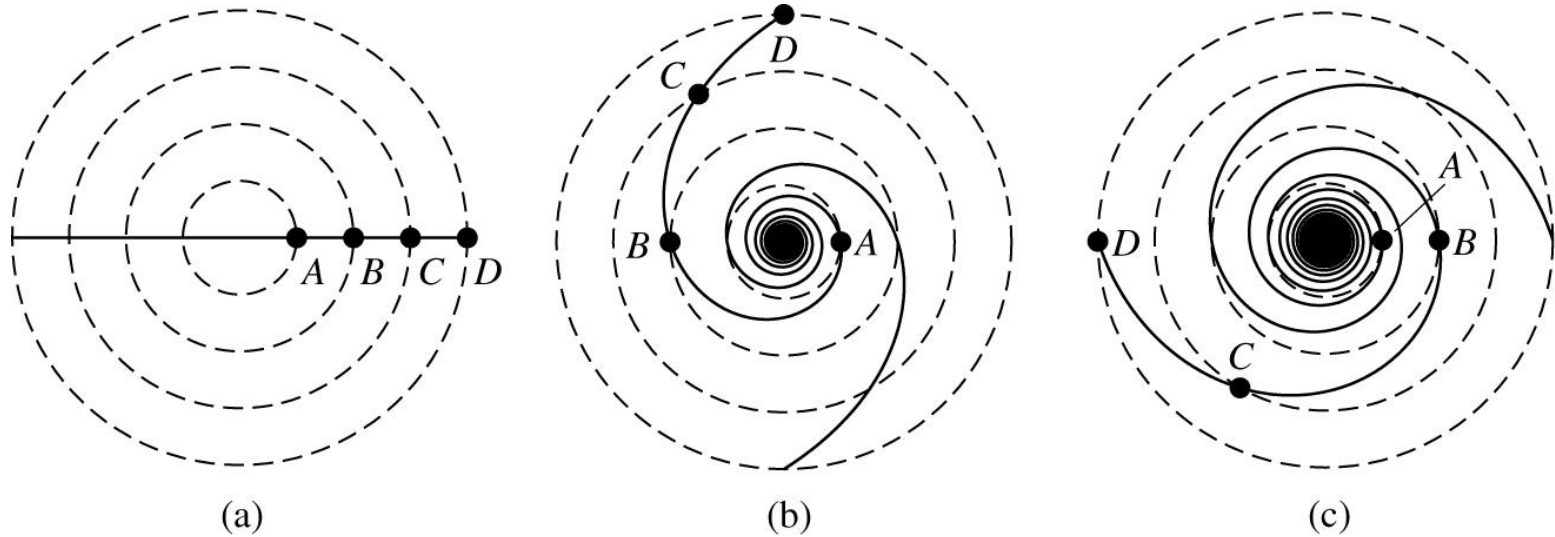
Trailing structure



Leading structure

Most galaxies show trailing structures in the sense that the tips of the spirals point in the opposite direction from the direction of rotation.

Spiral Arm Winding Problem



In spiral galaxies the stellar velocities of stars in the disk is approximately constant with radius. As a result, if the spiral arms were moving with the stars the arms would end up winding up within a few orbital periods.

Density-Wave Model of Spiral Arms

Most of the stars, gas and dust in our Galaxy rotate around the center of the galaxy at almost the same speed. Any rigid pattern of stars could not persist after some time.

The **density-wave model** posits that the spirals are actually density-waves that travel around the disk just like ripples on water. These waves move around the galaxy more slowly than do stars, dust and gas **near the center of the galaxy**.

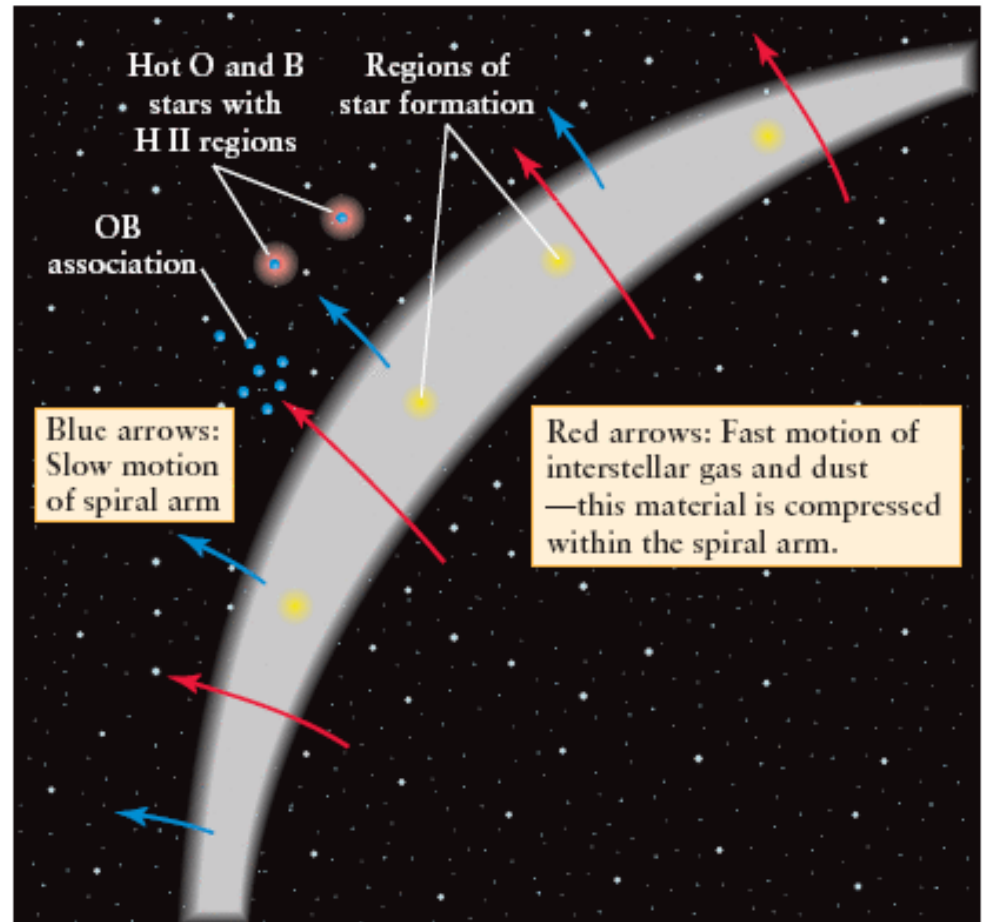
A density wave compresses the gases in the interstellar medium and this **leads eventually to star formation**.

Density-Wave Model of Spiral Arms

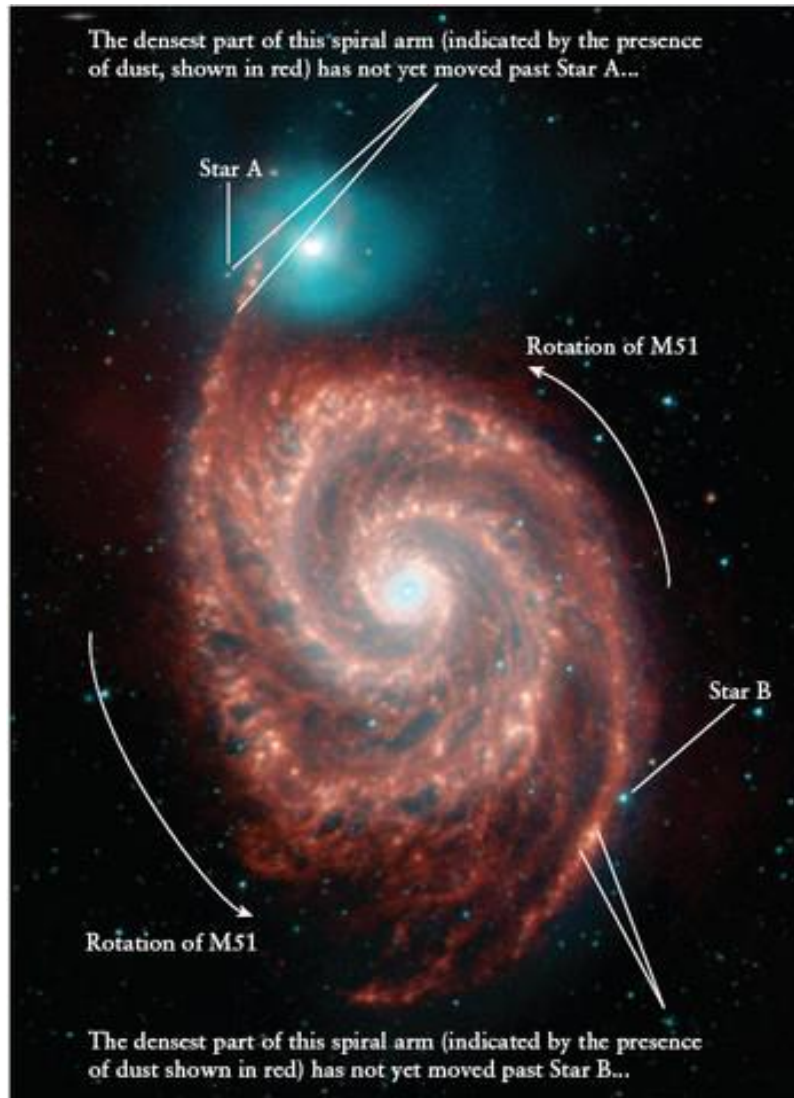
A **spiral arm** is a region where the density of material is higher than in the surrounding parts of a galaxy.

Interstellar matter moves around the galactic center rapidly (shown by the red arrows) and is **compressed as it passes through the slow-moving spiral arms** (whose motion is shown by the blue arrows).

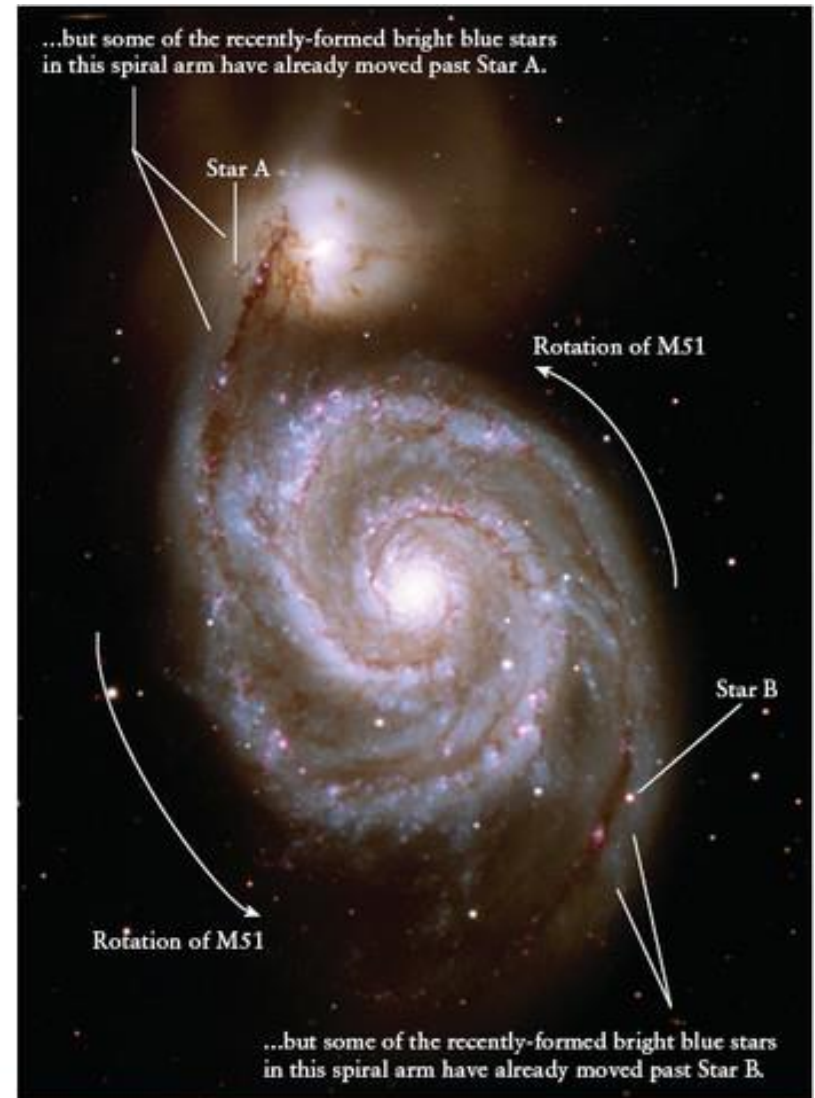
This **compression triggers star formation** in the interstellar matter, so that new stars appear on the “downstream” side of the densest part of the spiral arm.



Density-Wave Model of Spiral Arms



(a) An infrared view of M51 shows the locations of dust



(b) A visible-light view of M51 shows the locations of young stars

Self-Propagating Star-Formation Model of Spiral Arms



(a) Grand-design spiral galaxy

In the density wave model well defined spiral arms produce star formation.



(b) Flocculent spiral galaxy

In the self-propagating model chaotic looking spiral arms are produced by star formation.

Lin-Shu Density Wave Theory

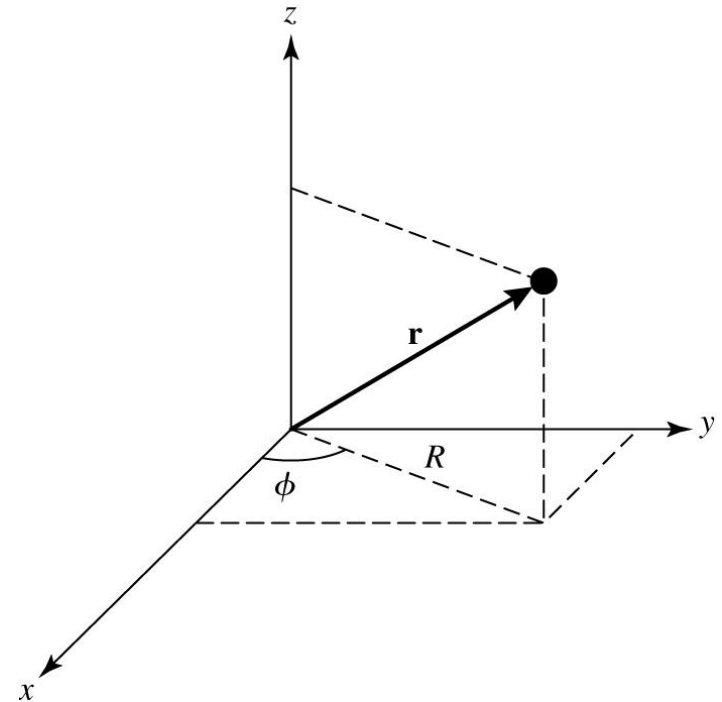
- Spiral structure arises from the presence of long-lived density waves
- Density Waves: regions in the galactic disk where the mass density of stars is $\sim 10\text{-}20\%$ higher than average.
- Density waves have different velocity than the stars, dust and gas.
- Density waves rotate at a global pattern speed Ω_{gp}
- Stars near the center have $\Omega_{\text{star}} > \Omega_{\text{gp}}$
- Lin-Shu theory can explain the presence of massive stars and HII regions throughout the spiral arms.

Equation of Motion of Star in Axially Symmetric Gravitational Field

$$M \frac{d^2 r}{dt^2} = F_g$$

Use cylindrical coordinates R, ϕ, z

The unperturbed orbit is a circle of radius $R = R_m$ and $z = 0$



Equation of Motion of Star in Axially Symmetric Gravitational Field

The orbit of the star with a small perturbation is:

$$\rho(t) = R(t) - R_m = A_R \sin(\kappa t)$$

$$z(t) = A_z \sin(\nu t + \zeta)$$

$$\chi(t) = \frac{2\Omega}{\kappa} A_R \cos(\kappa t)$$

Where $\rho(t)$ is the offset of the star in the radial direction from the unperturbed radius R_m

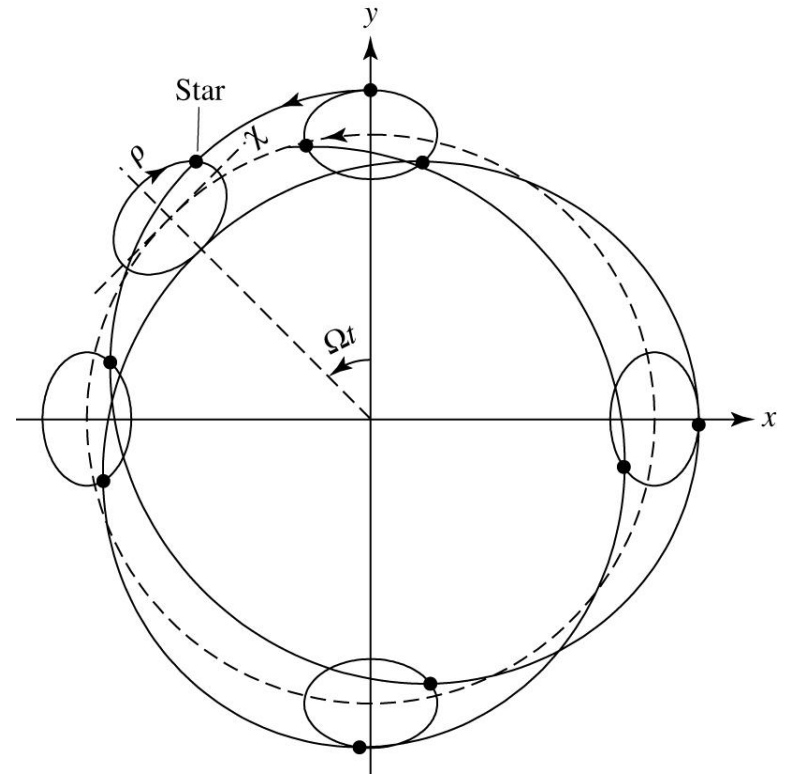
κ is the epicycle frequency

A_R is the amplitude of the epicycle

A_z is the amplitude of the oscillation in the z direction

$\chi(t)$ is the offset of the star in the azimuthal direction

Ω is the orbital angular speed of the unperturbed star around the galactic center



Equation of Motion of Star in Axially Symmetric Gravitational Field

The orbit of the star with a small perturbation is:

$$\rho(t) = R(t) - R_m = A_R \sin(\kappa t)$$

$$z(t) = A_z \sin(\nu t + \zeta)$$

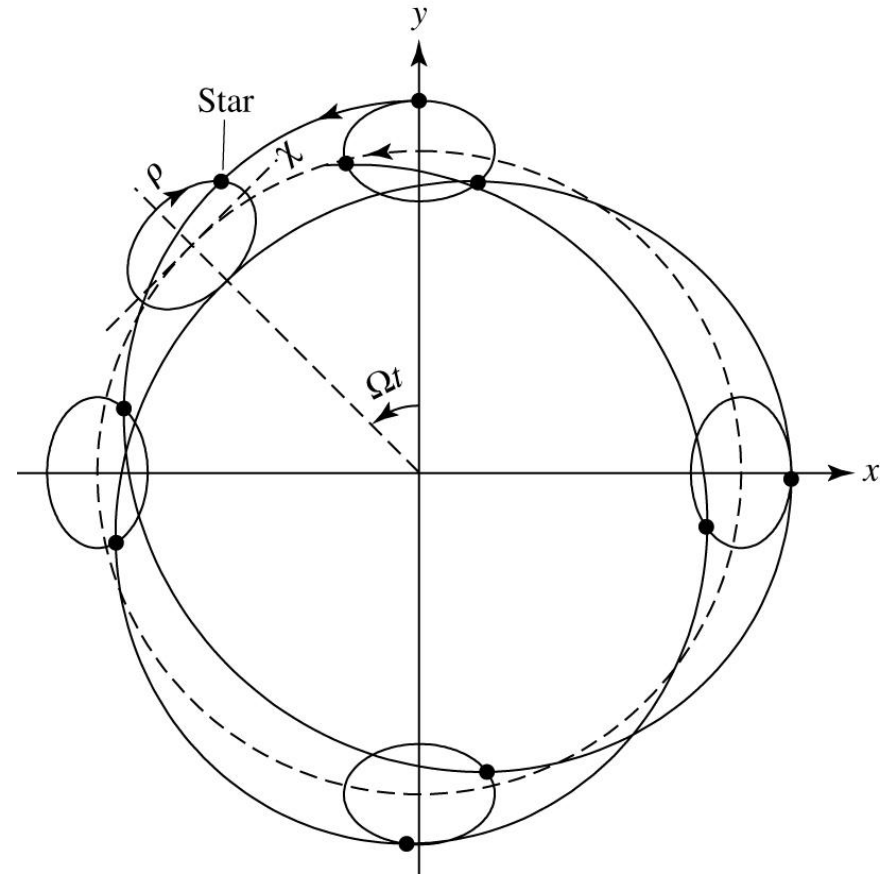
$$\chi(t) = \frac{2\Omega}{\kappa} A_R \cos(\kappa t)$$

$$\kappa^2 = \left. \frac{\partial^2 \Phi_{eff}}{\partial R^2} \right|_m$$

$$\nu^2 = \left. \frac{\partial^2 \Phi_{eff}}{\partial z^2} \right|_m$$

$$\Phi_{eff} = \Phi + \frac{J_z^2}{2R^2}$$

$$\Phi = \frac{U}{M}, \quad F_g = -\nabla U, \quad J_z = R^2 \dot{\phi}$$



Closed Orbits in Noninertial Frames

Ω = Orbital frequency of rotation of unperturbed star around GC

κ = the frequency of the epicycle

Period to go around 1 epicycle = $2\pi / \kappa$

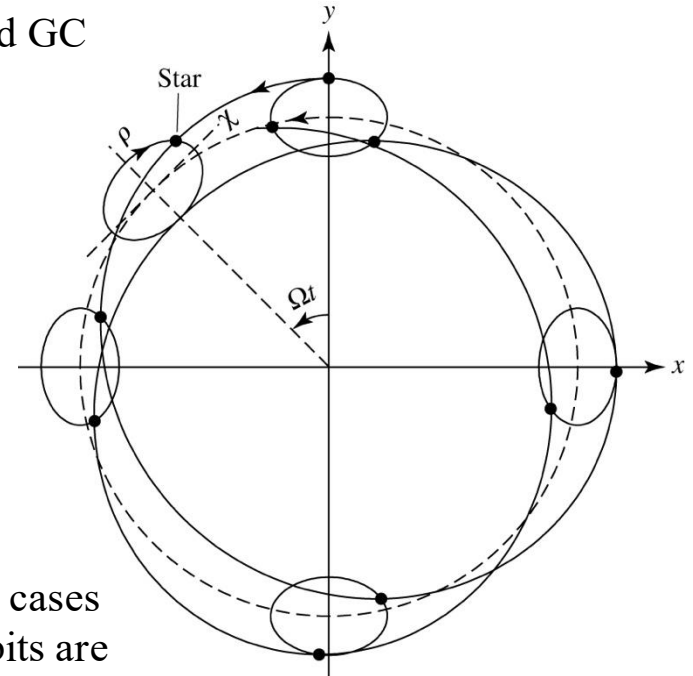
Period to go around 1 orbit = $2\pi / \Omega$

It follows that the number of times a star goes around the epicycle per orbit around the galactic center is:

$$n = (\text{Period of orbit} / \text{period of epicycle}) = \kappa / \Omega$$

If n is an integer the orbit will be closed. However, in most cases (like our Sun's) **in an inertial reference** frame the stars orbits are not closed.

However, **in a noninertial frame** orbiting with a local angular pattern speed of $\Omega_{lp} = \Omega$ the observer will see the star perform a closed epicycle.



Closed Orbits in Noninertial Frames

In general, we can obtain closed orbits **in noninertial frames** for angular pattern speeds that satisfy the following equation:

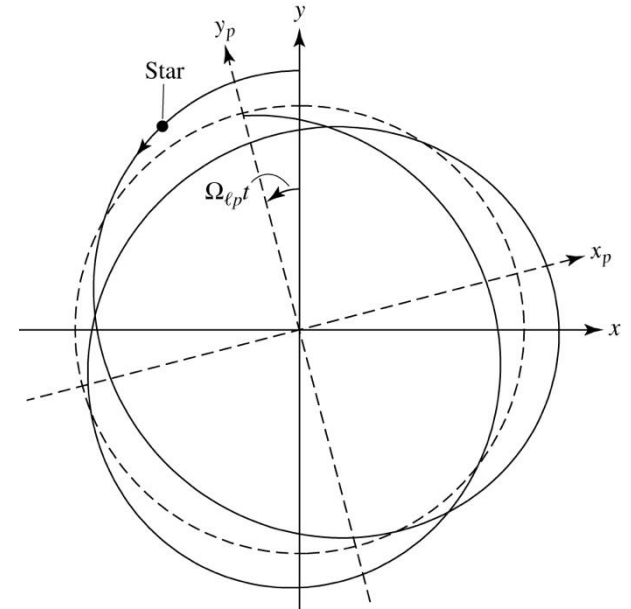
$$\Omega_{lp}(R) = \Omega(R) - \frac{n}{m}\kappa(R), \text{ where } n \text{ and } m \text{ are integers}$$

A closed orbit is obtained when after the star executes **n** orbits around the galactic center it executes **m** epicycle orbits and gets back to the same point.

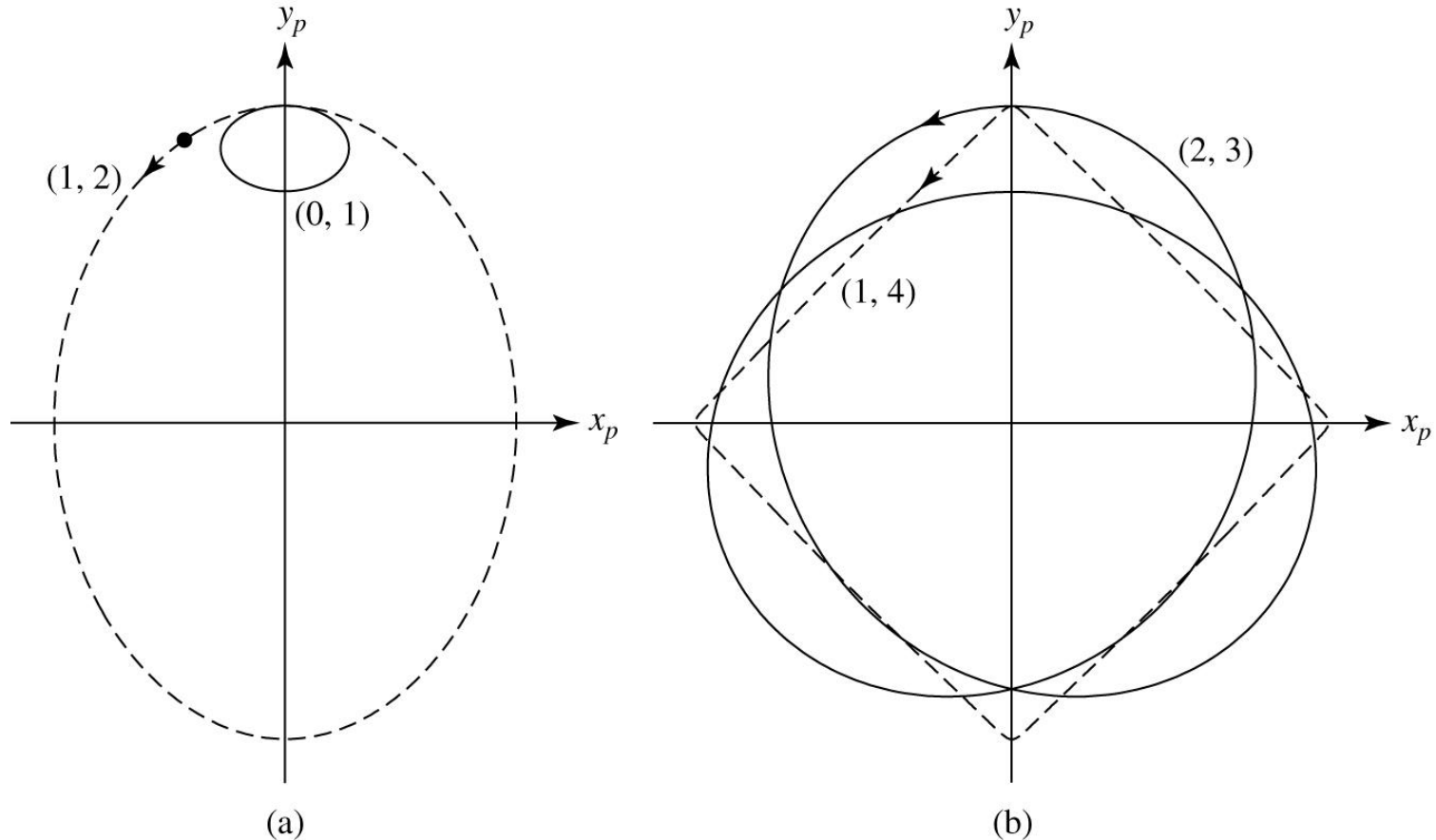
If you are rotating in a frame with a frequency $\Omega_{lp} = \Omega$ the epicycle will appear stationary. If you are rotating at a smaller frequency of $\Omega_{lp} < \Omega$ the epicycle will appear to rotate at a frequency of $\Omega - \Omega_{lp}$. n integer orbits will be executed in $P = 2\pi n / (\Omega - \Omega_{lp})$

The epicycle will execute m integer orbits in a period $P = 2\pi m / \kappa$. Equating the periods we obtain:

$$n / (\Omega - \Omega_{lp}) = m / \kappa$$



Closed Orbits in Noninertial Frames

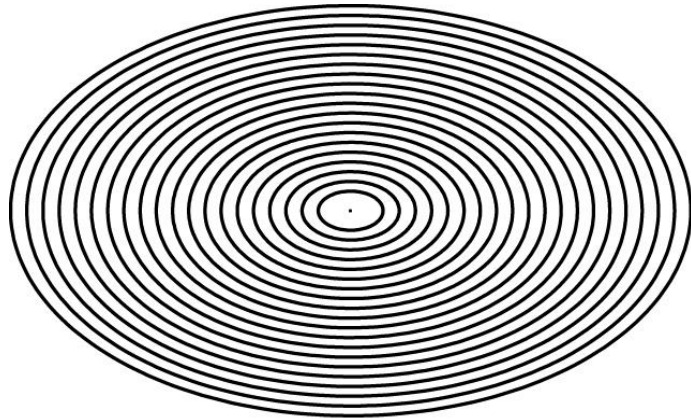


$n, m = (1, 2)$ dashed

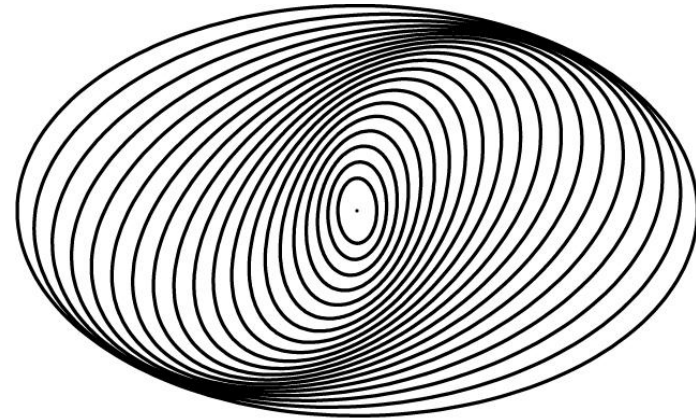
$n, m = (0, 1)$ solid (epicycle not rotating in noninertial frame)

n orbits around galactic center m orbits around epicycle

Closed Orbits in Noninertial Frames



(a)



(b)

Now imagine a large number of stars at various R all observed from a noninertial frame orbiting at a global angular pattern speed of $\Omega_{gp} = \Omega_{lp} = \Omega(R) - \kappa(R)/2$ (where $n = 1$, $m = 2$ and we assumed that Ω_{lp} does not vary with R)

An observer in the noninertial frame would see the pattern on the left. Slight offsets between the orbits of neighboring stars will result in the pattern on the right.

In a non-rotating system these patterns will appear to rotate at the global angular pattern speed of Ω_{gp}

Examples

Using the solar motion data given in chapter 24 of C&O estimate the amplitude of the Sun's excursion in the radial direction relative to a perfectly circular motion. Assume that the sun is currently at the midpoint of its oscillation.

Solution:

Examples

C&O 25.14: Using the solar motion data given in chapter 24 of C&O estimate the amplitude of the Sun's excursion in the radial direction relative to a perfectly circular motion. Assume that the sun is currently at the midpoint of its oscillation.

Solution:

The excursion in the radial direction is:

$$\rho(t) = R(t) - R_m = A_R \sin(\kappa t)$$

The radial velocity is just the derivative of $\rho(t)$:

$$u_r(t) = A_R \kappa \cos(\kappa t)$$

The radial velocity and amplitude of the Sun's excursion now are :

$$u_{\odot}(0) = A_R \kappa = 10 \text{ km s}^{-1} \rightarrow A_R = \frac{10 \text{ km s}^{-1}}{\kappa} = 250 \text{ pc}$$

Examples

C&O 25.16: Determine the epicycle axis ratio $\frac{\chi(t)}{\rho(t)}$ for a Keplerian orbit (a point mass orbiting a central, spherically symmetric mass). Hint Begin with Eq. (25.37)

Solution:

Examples

C&O 25.16: Determine the epicycle axis ratio $\frac{\chi(t)}{\rho(t)}$ for a Keplerian orbit (a point mass orbiting a central, spherically symmetric mass). Hint Begin with Eq. (25.37)

Solution:

$$\chi(t) = \frac{2\Omega_0}{\kappa_0} A_R \cos \kappa t \qquad \rho(t) = A_R \sin \kappa t$$

$$\frac{\chi_{max}}{\rho_{max}} = \frac{\frac{2\Omega_0}{\kappa_0} A_R}{A_R} = \frac{2\Omega_0}{\kappa_0}$$

$$\kappa^2 = \frac{2\Theta_0}{R_0} \left[\left. \frac{\partial \Theta}{\partial R} \right|_{R_0} + \frac{\Theta_0}{R_0} \right] \qquad \Omega_0 = \frac{\Theta_0}{R_0}$$

Examples

C&O 25.16: Determine the epicycle axis ratio $\frac{\chi(t)}{\rho(t)}$ for a Keplerian orbit (a point mass orbiting a central, spherically symmetric mass). Hint Begin with Eq. (25.37)

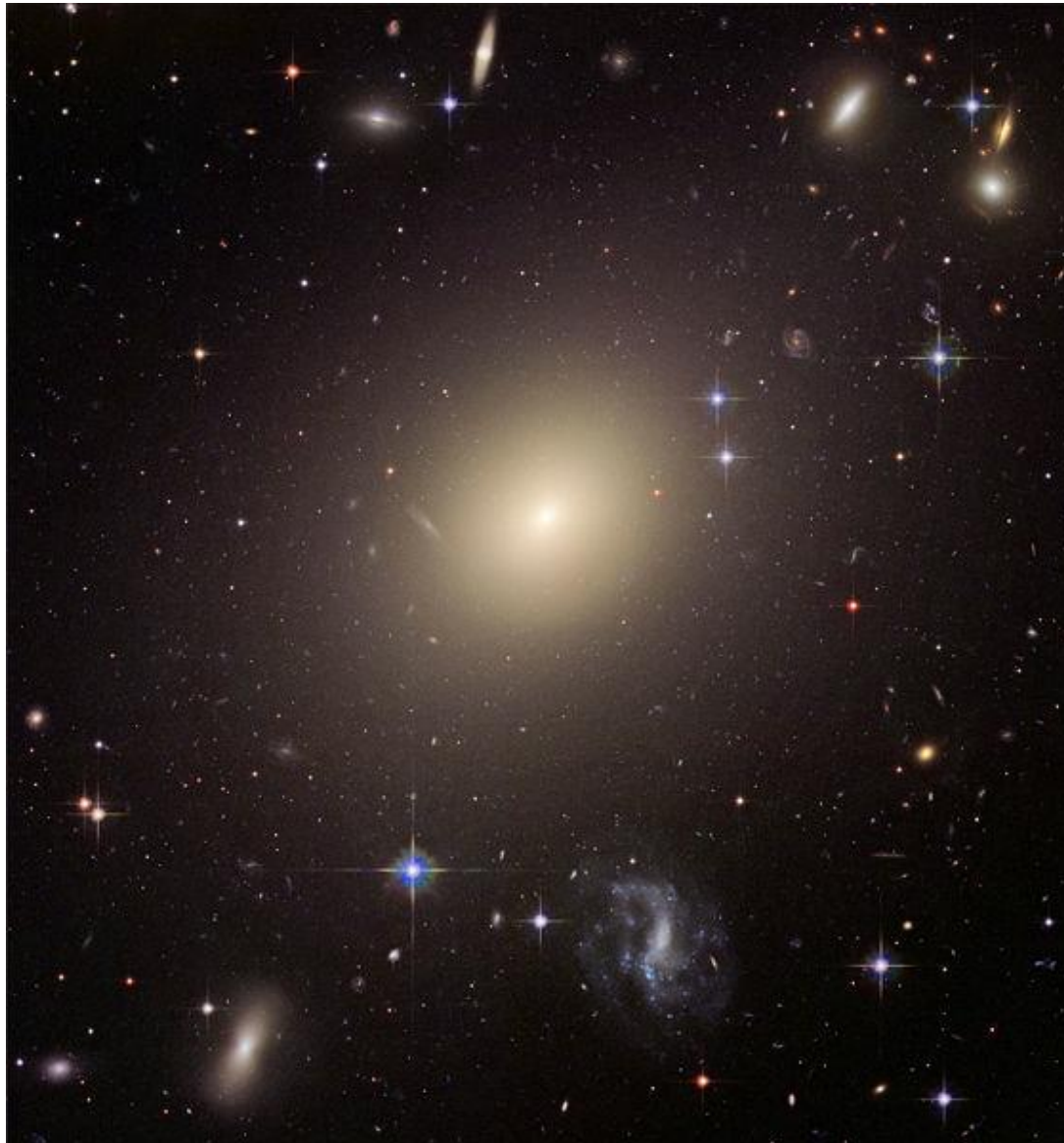
Solution:

$$\frac{\chi_{max}}{\rho_{max}} = \frac{2\Omega_0}{\kappa_0} = \frac{\frac{2\Theta_0}{R_0}}{\left(\frac{2\Theta_0}{R_0}\right)^{1/2} \left[\frac{\partial \Theta}{\partial R} \Big|_{R_0} + \frac{\Theta_0}{R_0} \right]^{1/2}} = \frac{\left(\frac{2\Theta_0}{R_0}\right)^{1/2}}{\left[\frac{(GM)^{1/2}}{2R_0^{3/2}} \right]^{1/2}} = \frac{\left[\frac{2(GM)^{1/2}}{R_0^{3/2}} \right]^{1/2}}{\left[\frac{(GM)^{1/2}}{2R_0^{3/2}} \right]^{1/2}} = 2$$

Where we used for Keplerian motion: $\Theta^2 = \frac{GM}{R} \rightarrow \Theta = \left(\frac{GM}{R}\right)^{1/2} \rightarrow$

$$\frac{\partial \Theta}{\partial R} \Big|_{R_0} = (GM)^{1/2} \left(-\frac{1}{2} \right) \frac{1}{R_0^{3/2}} = -\frac{(GM)^{1/2}}{2R_0^{3/2}} \text{ and } \frac{\Theta_0}{R_0} = \frac{\left(\frac{GM}{R_0}\right)^{1/2}}{R_0} = \frac{(GM)^{1/2}}{R_0^{3/2}}$$

Elliptical Galaxies



The giant elliptical galaxy ESO 325-G004

TABLE 25.3 Characteristic Data for cD, Elliptical, and Lenticular Galaxies.

	cD	E	S0/SB0
M_B	-22 to -25	-15 to -23	-17 to -22
M ($M_\odot$)	10^{13} – 10^{14}	10^8 – 10^{13}	10^{10} – 10^{12}
Diameter (D_{25} , kpc)	300–1000	1–200	10–100
$\langle M/L_B \rangle$ ($M_\odot/L_\odot$)	> 100	10–100	~ 10
$\langle S_N \rangle$	~ 15	~ 5	~ 5

TABLE 25.4 Characteristic Data for Dwarf Elliptical, Dwarf Spheroidal, and Blue Compact Dwarf Galaxies.

	dE	dSph	BCD
M_B	-13 to -19	-8 to -15	-14 to -17
M ($M_\odot$)	10^7 – 10^9	10^7 – 10^8	~ 10^9
Diameter (D_{25} , kpc)	1–10	0.1–0.5	< 3
$\langle M/L_B \rangle$ ($M_\odot/L_\odot$)	~ 10	5–100	0.1–10
$\langle S_N \rangle$	4.8 ± 1.0	—	—

Tables 25.3 and 25.4 summarize some of the characteristics of early-type galaxies. Lenticular galaxies have also been included in the list, since they appear to have much in common with many ellipticals.

Dust and Gas in Ellipticals

Gas and dust are present in ellipticals but at significantly diminished levels compared to spirals.

It is thought that ellipticals may have lost gas by:

- Ram pressure stripping
- Supernovae
- AGN winds and jets

- As in spirals the metallicity in ellipticals increases with their luminosity

The Faber-Jackson Relation in Elliptical Galaxies

1. Assume a constant mass-to-light ratio: $\frac{M}{L} = c_1$ (1)
2. Assume the virial mass equation is valid for elliptical galaxies (R is the galaxy radius) : $M = \frac{5R\sigma_r^2}{G}$ (2)

$$(1) \wedge (2) \Rightarrow L = \frac{5R\sigma_r^2}{Gc_1} \quad (3)$$

3. Assume ellipticals have the same surface brightness

$$I \propto \frac{L}{R^2} = c_2 \Rightarrow R = \frac{L^{1/2}}{c_2^{1/2}} \quad (4)$$

$$(3) \wedge (4) \Rightarrow L = \frac{5\sigma_r^2}{Gc_1} \frac{L^{1/2}}{c_2^{1/2}} \Rightarrow L = \frac{25\sigma_r^4}{c_1^2 c_2 G} \Rightarrow L \propto \sigma_r^4$$

This is the Faber-Jackson relation for elliptical galaxies

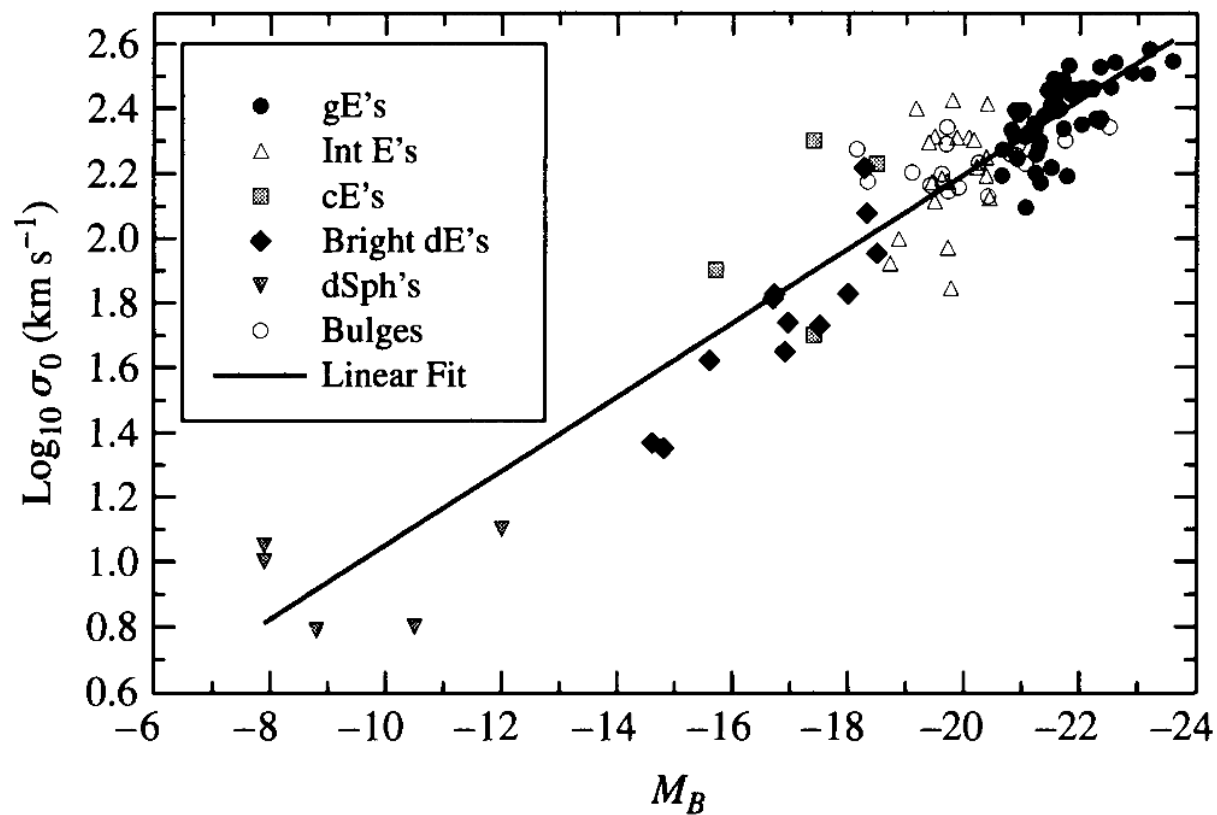


FIGURE 25.33 The Faber–Jackson relation represents a correlation between central velocity dispersion and luminosity. The relation is a consequence of the virial theorem. (Data from Bender et al., *Ap. J.*, 399, 462, 1992.)

Estimating the distances to Elliptical Galaxies

There is significant scatter in the $L \propto \sigma_r^4$ relation. A tighter fit to the data is given by the relation

$$L \propto \sigma_r^{2.65} r_e^{0.65}$$

Where r_e is the **effective radius** within which half of the light of the galaxy resides.

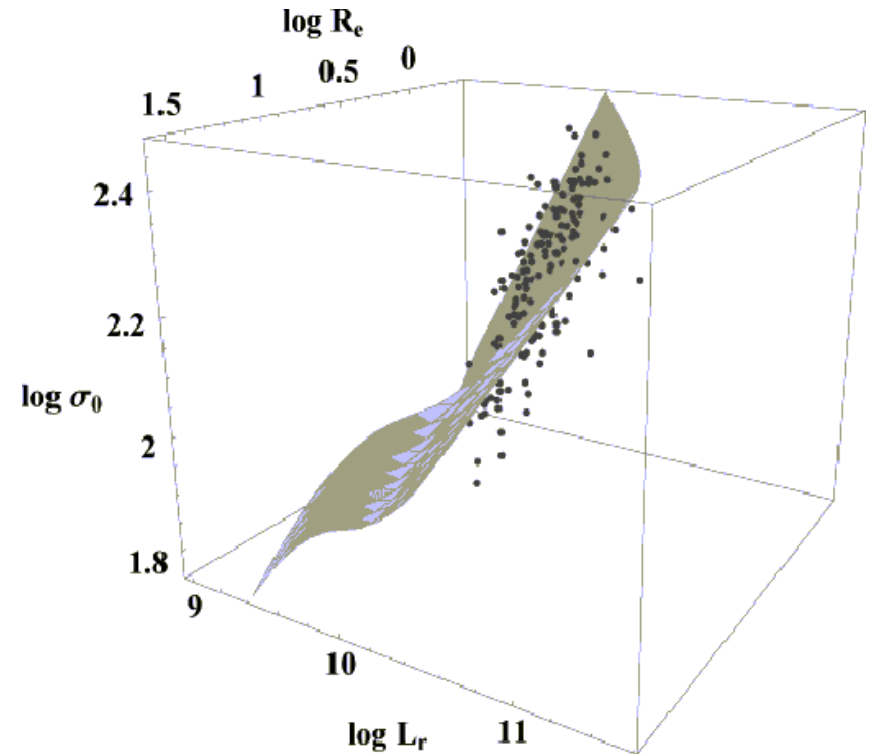
The fundamental plane is an empirical relation between the effective **size**, the average surface **brightness** and the central **velocity dispersion of the central stars** found in elliptical galaxies.

If one measures these three quantities for many elliptical galaxies and plots them they find that the points fall on a plane called the fundamental plane.

Estimating the distances to Elliptical Galaxies

Having found the fundamental plane for nearby galaxies one can estimate the distance to remote elliptical galaxies:

First one measures the central velocity dispersion of the stars and the average surface brightness. Then using the fundamental plane one can infer the effective radius of the galaxy and use the small angle formula to estimate the distance.



Galaxy Luminosity Function

$\Phi(L)dL$ is the number of galaxies per unit volume with luminosities between L and $L+dL$

$\Phi(M)dM$ is the number of galaxies per unit volume with absolute magnitude between M and $M+dM$

$\int \Phi(L)dL$ is the number density of all galaxies

Schechter Luminosity Function

$$\Phi(L) = \left(\frac{\Phi^*}{L^*} \right) \left(\frac{L}{L^*} \right)^a e^{-\frac{L}{L^*}}$$

Φ^* is a normalization factor which defines the overall density of galaxies (number per cubic Mpc)

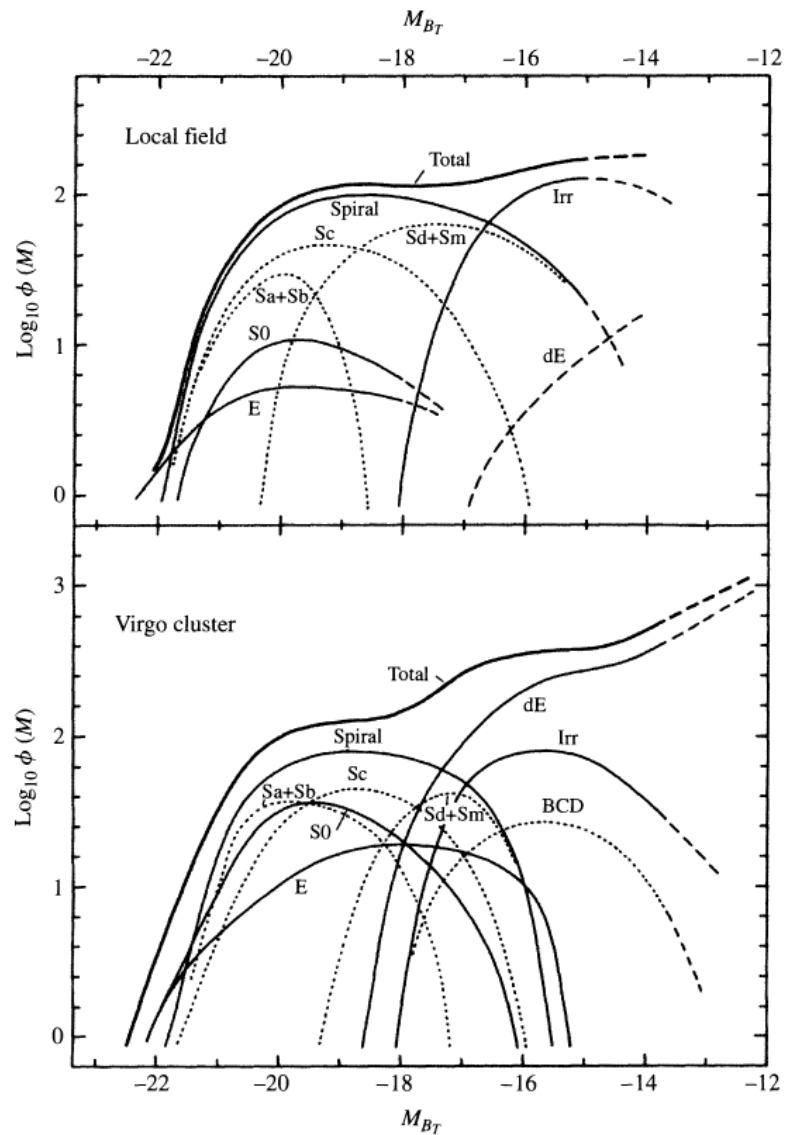
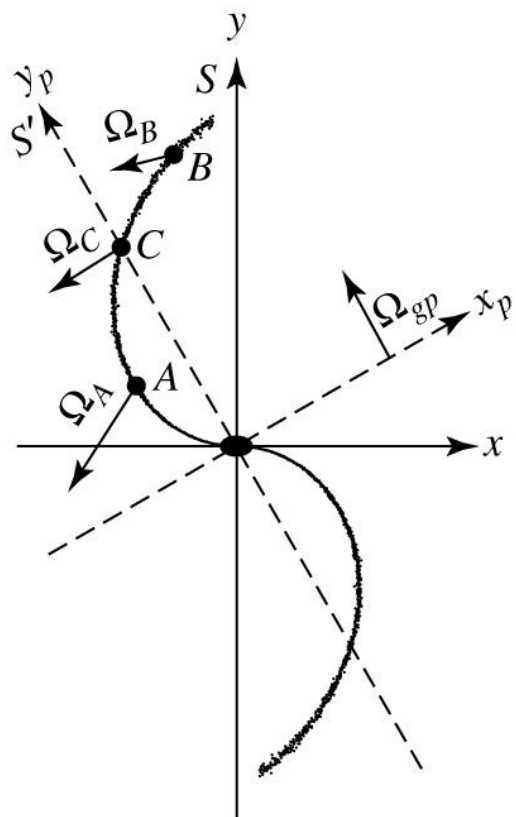
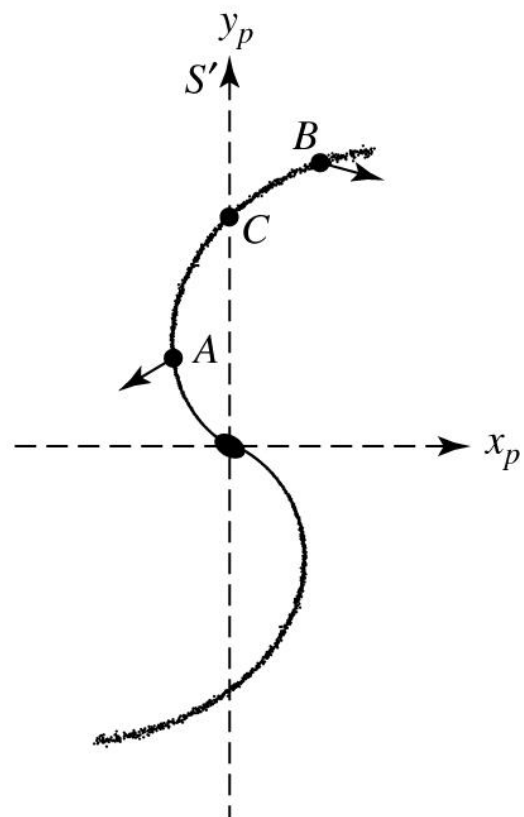


FIGURE 25.36 Luminosity functions for two samples of galaxies. The top panel is based on a sample of galaxies near the Milky Way Galaxy, and the bottom panel is for galaxies located in the Virgo cluster of galaxies. The zero point of the $\log_{10} \phi(M)$ axis is arbitrary. (Figure adapted from Binggeli, Sandage, and Tammann, *Annu. Rev. Astron. Astrophys.*, 26, 509, 1988. Reproduced with permission from the *Annual Review of Astronomy and Astrophysics*, Volume 26, ©1988 by Annual



(a)



(b)

