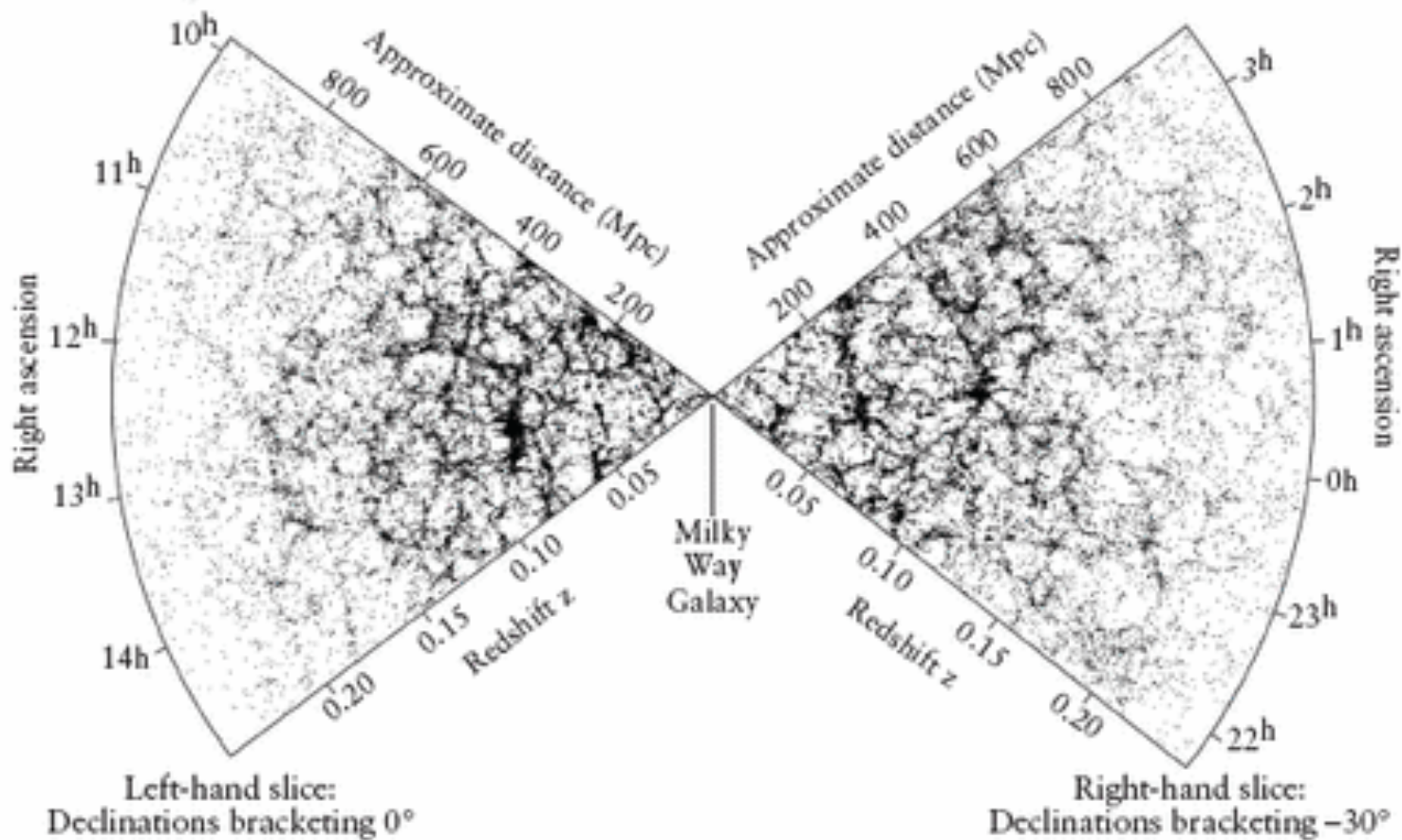


The Structure of the Universe



(a) The 2dF galaxy survey

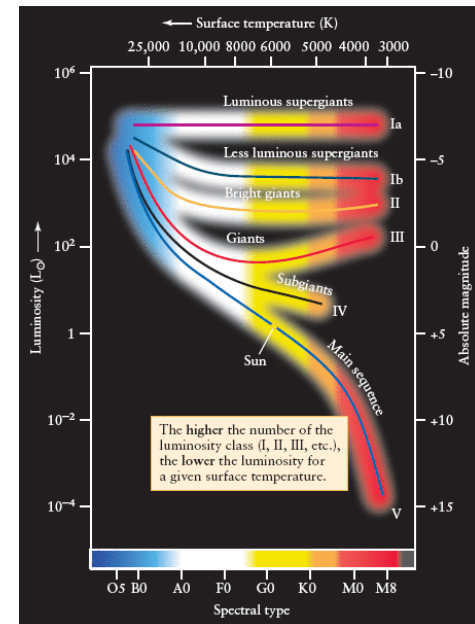
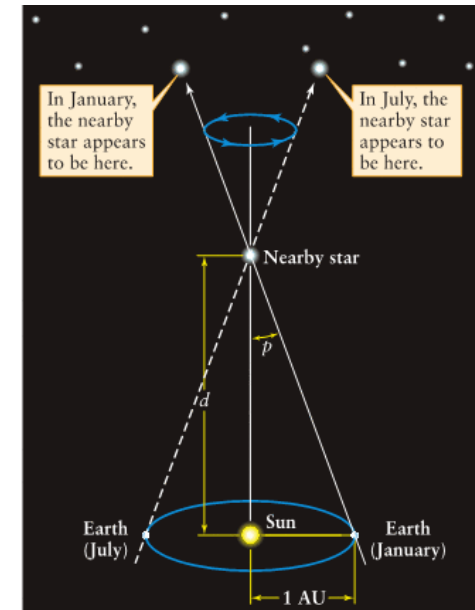
Measuring the Distances to Galaxies

Parallax methods cannot be used to measure the distances to galaxies since they can only be used to measure objects relatively nearby.

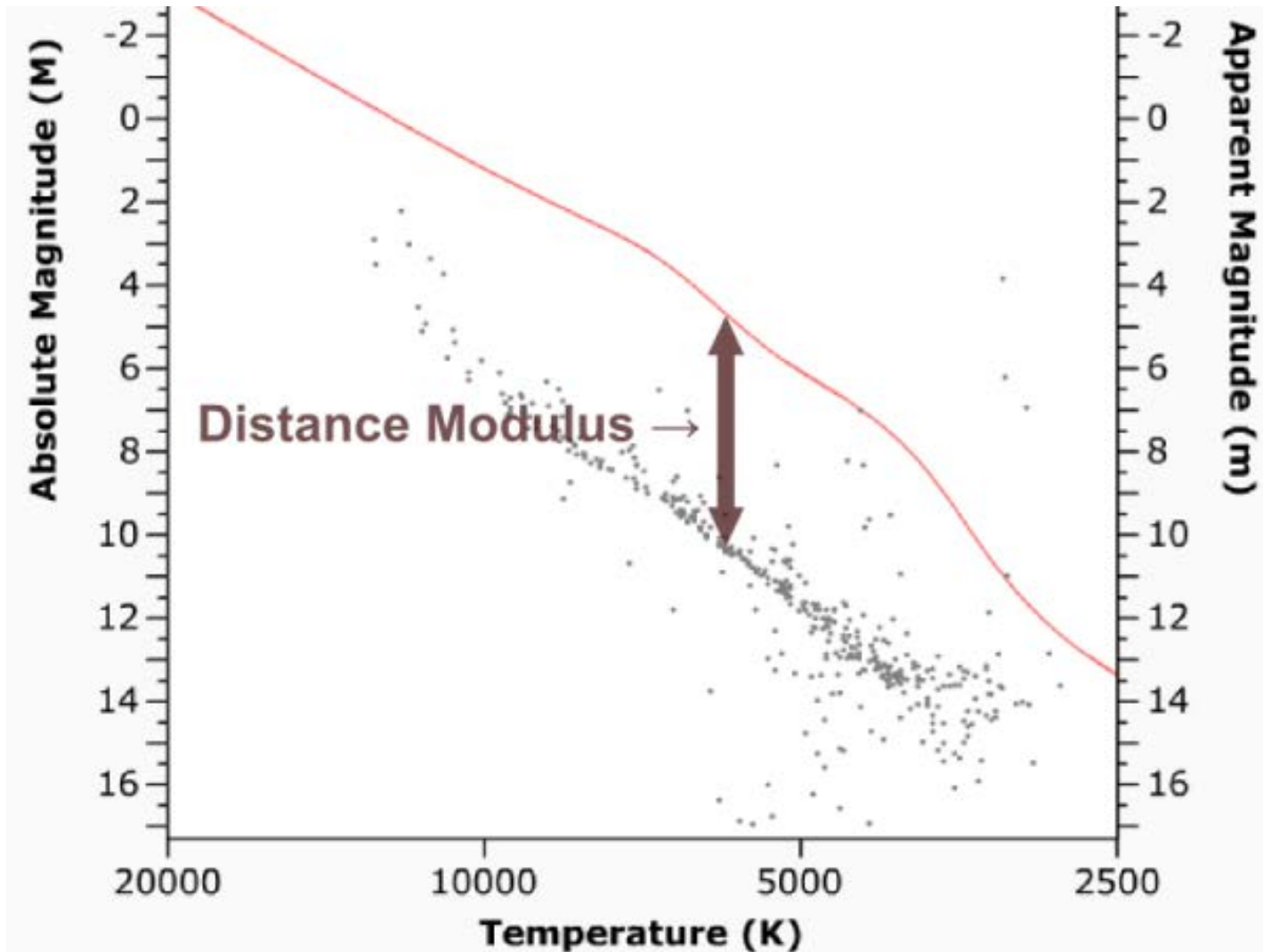
Observations by the GAIA satellite permit measurements of parallax angles down to $200 \mu \text{ arcsec}$ and distances to 5 kpc

$$d = (1/p) \text{ parsec, } p \text{ in arcsec.}$$

Spectroscopic parallax can measure objects accurately up to about 10 kpc . (From a stellar spectrum obtain the spectral type and luminosity class. Then use the HR diagram to infer luminosity or absolute magnitude. Combine absolute and apparent magnitude to determine distance using : $m - M = 5 \log d - 5$



Main Sequence Fitting

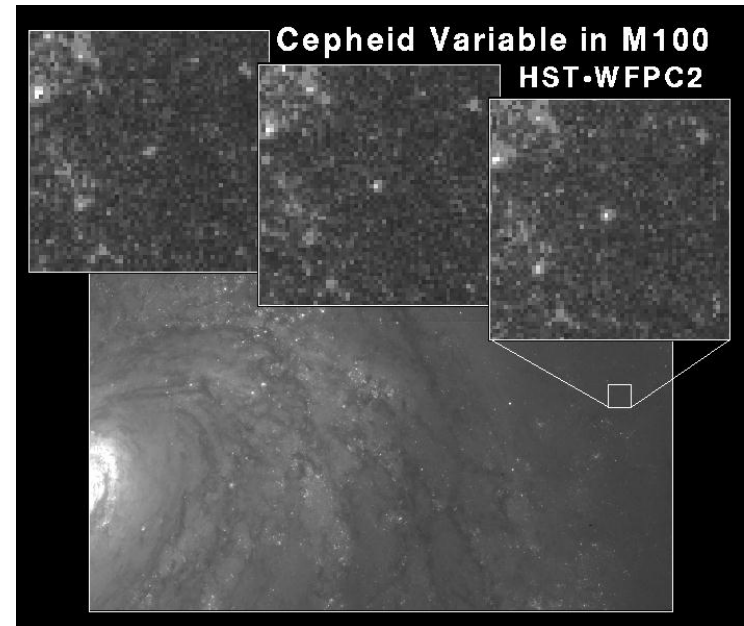


Variable Stars: Cepheid Variables

Pulsating supergiant stars that exhibit rapid brightening followed by gradual dimming with periods ranging from a few to a hundred days **are called Cepheid variables.**

The pulsations of the surfaces of Cepheids are caused by an imbalance between the power generated in the core and power radiated from the surface.

The first Cepheid variable discovered was δ Cephei.



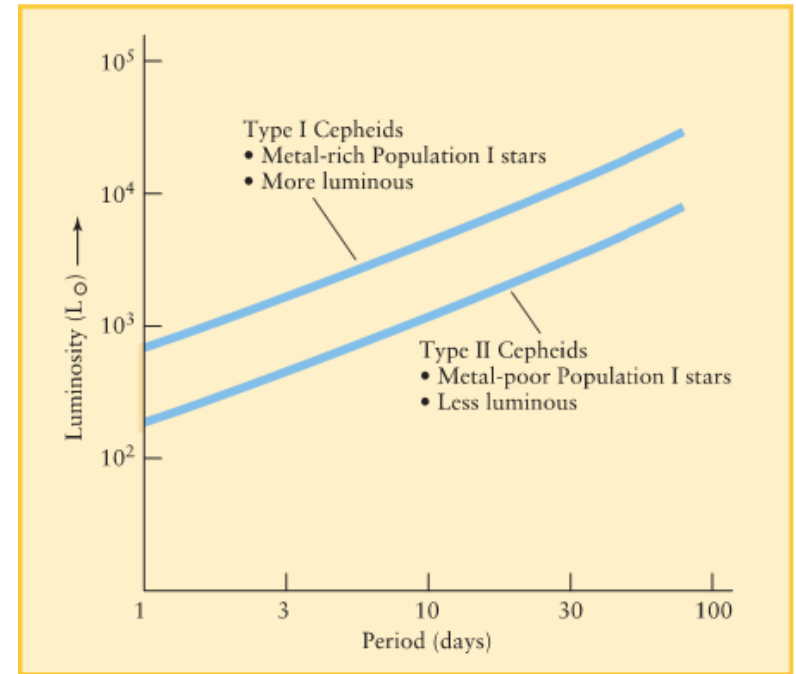
Period-Luminosity Relations of Cepheids

A measurement of the apparent magnitude (or flux) of an object combined with its absolute magnitude (or luminosity) that is inferred from the **Period-Luminosity relation** of Cepheids provides the distance to the object.

For classical Type-I Cepheids Henrietta Leavitt discovered:

$$M_V = -3.53 \log P_d - 2.13 + 2.13(B - V)$$

(period in days)



Period-Luminosity Relations for Cepheids. The greater the average luminosity of a Cepheid variable, the longer its period. There are two distinct period-luminosity relations—one for Type I Classical Cepheids and one for Type II Cepheids.

Distance to galaxy IC 4182

Example: Galaxy IC 4182

A metal rich Cepheid variable in IC 4182 has a period of 42 days. The period-luminosity relation gives a luminosity of $33,000L_{\odot}$ which corresponds to an absolute magnitude of -6.5.

Observations indicate the apparent magnitude of this Cepheid to be $m = +22.0$. The distance to the Cepheid variable in IC 4182 is:

$$m - M = 5 \log d - 5$$

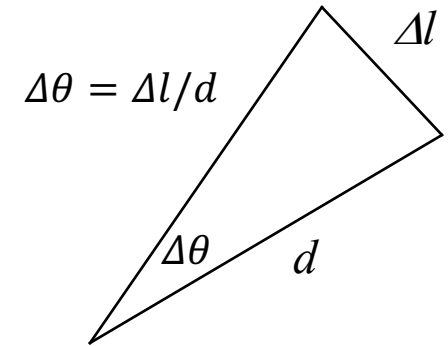
$$d = 10^{(m-M+5)/5} \text{ parsec} = 5 \times 10^6 \text{ parsec}$$

$$M_{\langle V \rangle} = -3.53 \log_{10} P_d - 2.13 + 2.13(B - V)$$

Expanding Supernovae as Distance Indicators

$$\omega = \frac{\Delta\theta}{\Delta t} \text{ (angular expansion rate) (1)}$$

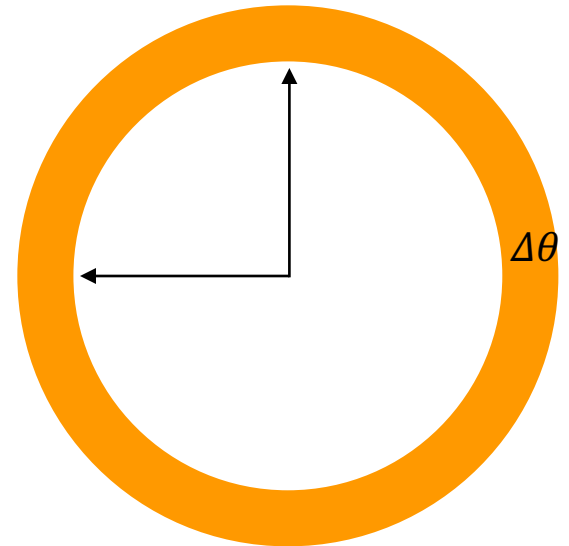
$$v_{ej} = \frac{\Delta l}{\Delta t} = d \frac{\Delta\theta}{\Delta t} \text{ (ejection velocity). (2)}$$



Assuming spherical ejection:

$$v_{ej} = \frac{\Delta\lambda}{\lambda} c \text{ (3)}$$

$$(1) \wedge (2) \wedge (3) \Rightarrow d = \frac{\Delta\lambda}{\lambda} c \frac{1}{\omega}$$



Measuring the Distances to Galaxies

Standard Candles: Objects of known luminosity.

1.) *Cepheid Variables* : Supergiant pulsating stars where their periods are a function of their average luminosity. Can be used to determine distances up to 100 Mly.

2.) *RR Lyrae Variables* : Pulsating horizontal branch stars of spectral class A with a period of less than a day. They are old, relatively low mass, metal-poor Population II stars.

Can be used to determine distances up to 300,000 ly.

3.) Expanding Supernovae as distance indicators

4.) *SN Type Ia*: these supernovae occur when a white dwarf in a close binary system accretes enough matter from its companion to blow itself apart when $M > 1.4 M_{\odot}$. Their spectra lack hydrogen emission but contain a strong Si absorption line.

As a first approximation the blue and visual absolute magnitudes of Type Ia's are :

$M_B \sim M_V \sim -19.3 \pm 0.03$ at maximum.

A large rate of decay of light-curve of Type Ia $\rightarrow$ low luminosity

Can be used to determine distances up to 3×10^9 ly.

Measuring distances to Galaxies with SN Type Ia



(a) M100 in March 2002

R I **V** U X G



(b) M100 in February 2006, showing Supernova 2006X

R I **V** U X G

VLT images of M100 (a) before and (b) after a Type Ia supernova exploded within the galaxy in 2006. The distance to M100 (~ 55 Mly) is also known from observations of Cepheid variables, so this particular supernova can help calibrate Type Ia supernovae as distance indicators.

Estimating the distances to Spiral Galaxies

Tully and Fischer found that the width of the hydrogen 21-cm line of a spiral galaxy is related to its luminosity. This is the **Tully-Fisher relation: $L \propto v_{max}^4$** .

The reason is that the faster a galaxy rotates the broader the 21 cm line becomes because of Doppler shifts of the receding and approaching sides of the galaxy.

Galaxies with faster rotational speeds are more massive according to Newton's form of Kepler's third law and the more massive galaxies contain more stars and are therefore more luminous. **Consequently, the width of a galaxy's 21-cm line is directly related to its luminosity.**

The Tully-Fisher relation can be used to determine the distance to spiral galaxies up to 100 Mpc.

Estimating the distances to Elliptical Galaxies

The fundamental plane is an empirical relation between the effective **size**, the average surface **brightness** and the central **velocity dispersion of the central stars** found in elliptical galaxies.

$$L \propto \sigma_r^{2.65} r_e^{0.65}$$

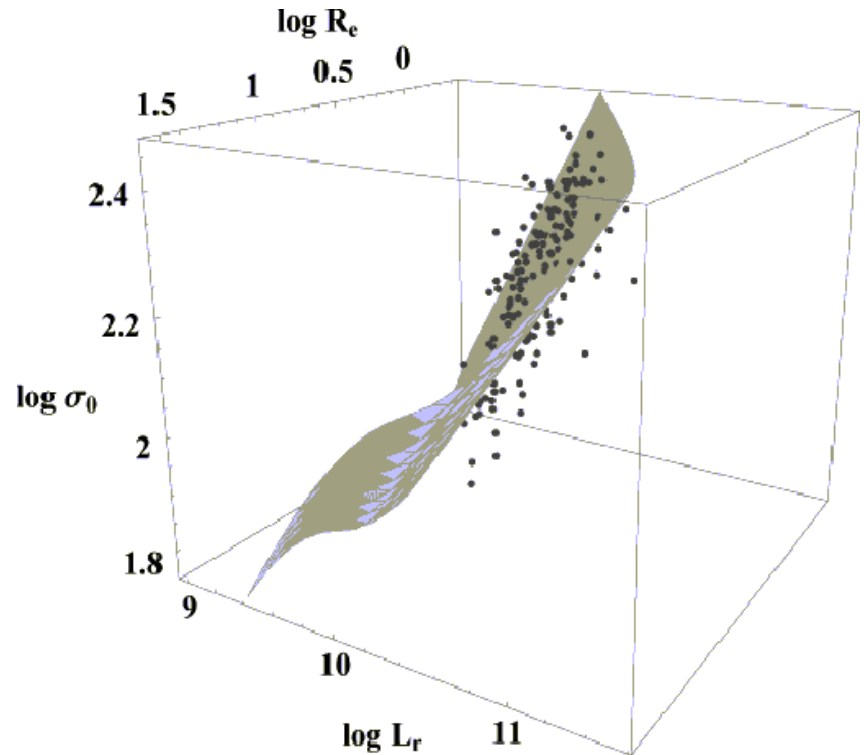
Where r_e is the **effective radius** within which half of the light of the galaxy resides

If one measures these three quantities for many elliptical galaxies and plots them, they find that the points fall on a plane called the **fundamental plane**.

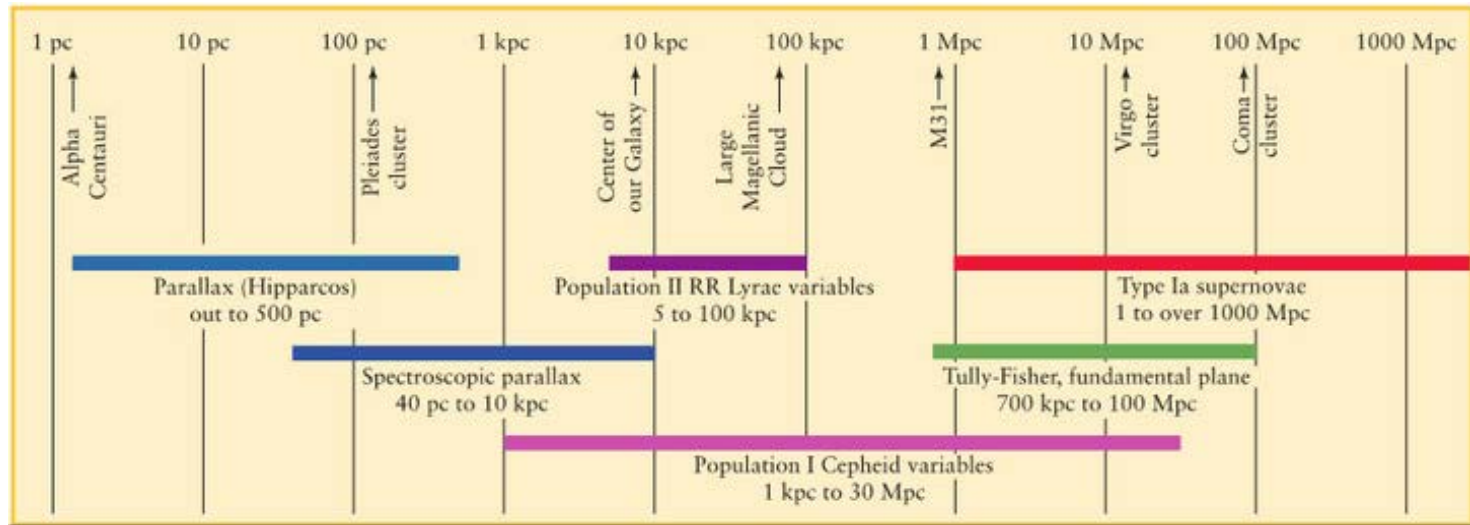
Estimating the distances to Elliptical Galaxies

Having found the fundamental plane for nearby galaxies one can estimate the distance to remote elliptical galaxies:

First one measures the velocity dispersion of the stars and the average surface brightness. Then using the fundamental plane one can infer the actual size of the galaxy and use the small angle formula to estimate the distance.



Distance Ladder



Astronomers use a sequence of methods to determine the distance to very remote objects. This approach is called the **distance ladder.**

Objects are found where their distances can be measured both with the accurate parallax method and the less accurate spectroscopic parallax and calibrate the spectroscopic method so it can be used at larger distances.

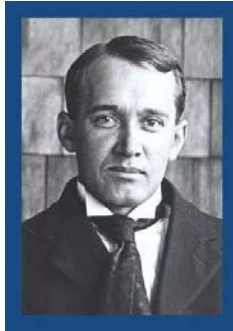
Next objects are found where their distance can be measured both with the spectroscopic parallax method and with Cepheid variables and calibrate the luminosities of the Cepheid variables and so on.

Expanding Universe

In the early 1900's it was Vesto Slipher from Lowell Observatory who first found that the spectra of most “nebulae” were redshifted. At the time it was not known how far these nebulae were and whether they were galactic or extra-galactic.



Lowell Observatory



Vesto Slipher



Lemaitre

In 1925 Hubble was able to determine the distance to M31 using Cepheid variables (and the L-P relation discovered by Henrietta Leavitt).

In Lemaitre (1927) first derived a linear relation between velocity and distance to galaxies.

Redshift

$$\frac{\lambda_{obs} - \lambda_0}{\lambda_0} = z$$

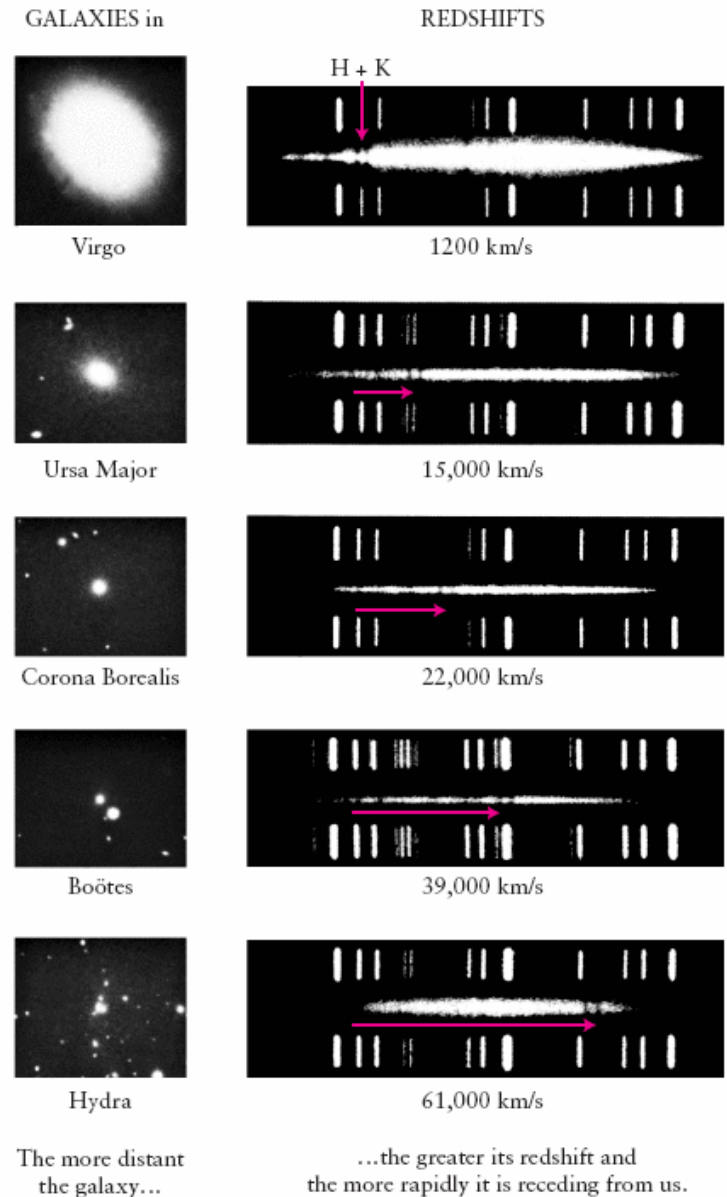
λ_{obs} = the observed wavelength

λ_0 = the rest-frame wavelength

z = the redshift of the object

Figure: Each galaxy's spectrum is a bright band with dark absorption lines.

The bright lines above and below it are a comparison spectrum taken on Earth. The horizontal red arrows show how much the H and K lines of singly ionized calcium are redshifted in each galaxy's spectrum. Below each spectrum is the recessional velocity calculated from the redshift.



The Hubble - Lemaitre Law

The Hubble - Lemaitre Law:

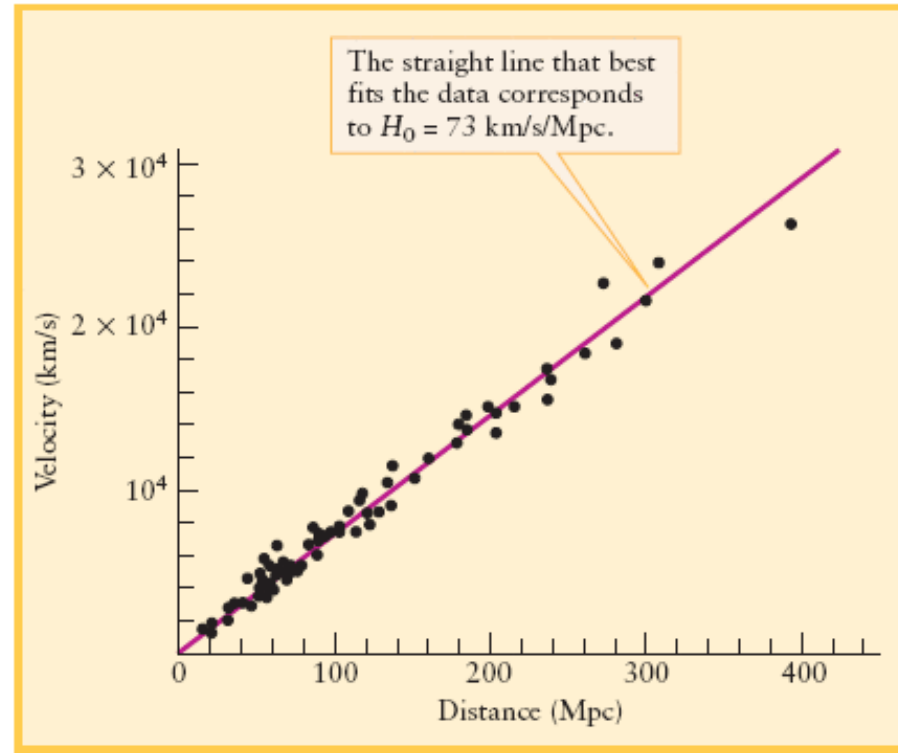
$$v = H_0 d$$

v = recessional velocity of a galaxy, H_0
= Hubble constant

d = comoving radial distance to the
galaxy

$$H_0 = 70 \pm 3 \text{ km/s/Mpc}$$

Note that this is not a real velocity as in
the Doppler effect but the apparent
velocity due to the expansion of space.
For low redshift $z \sim v/c$



Hubble measured the recession velocities (from redshifts) and distances to galaxies (using Cepheid variable stars) and found that the more distant the galaxy the greater its recession velocity.

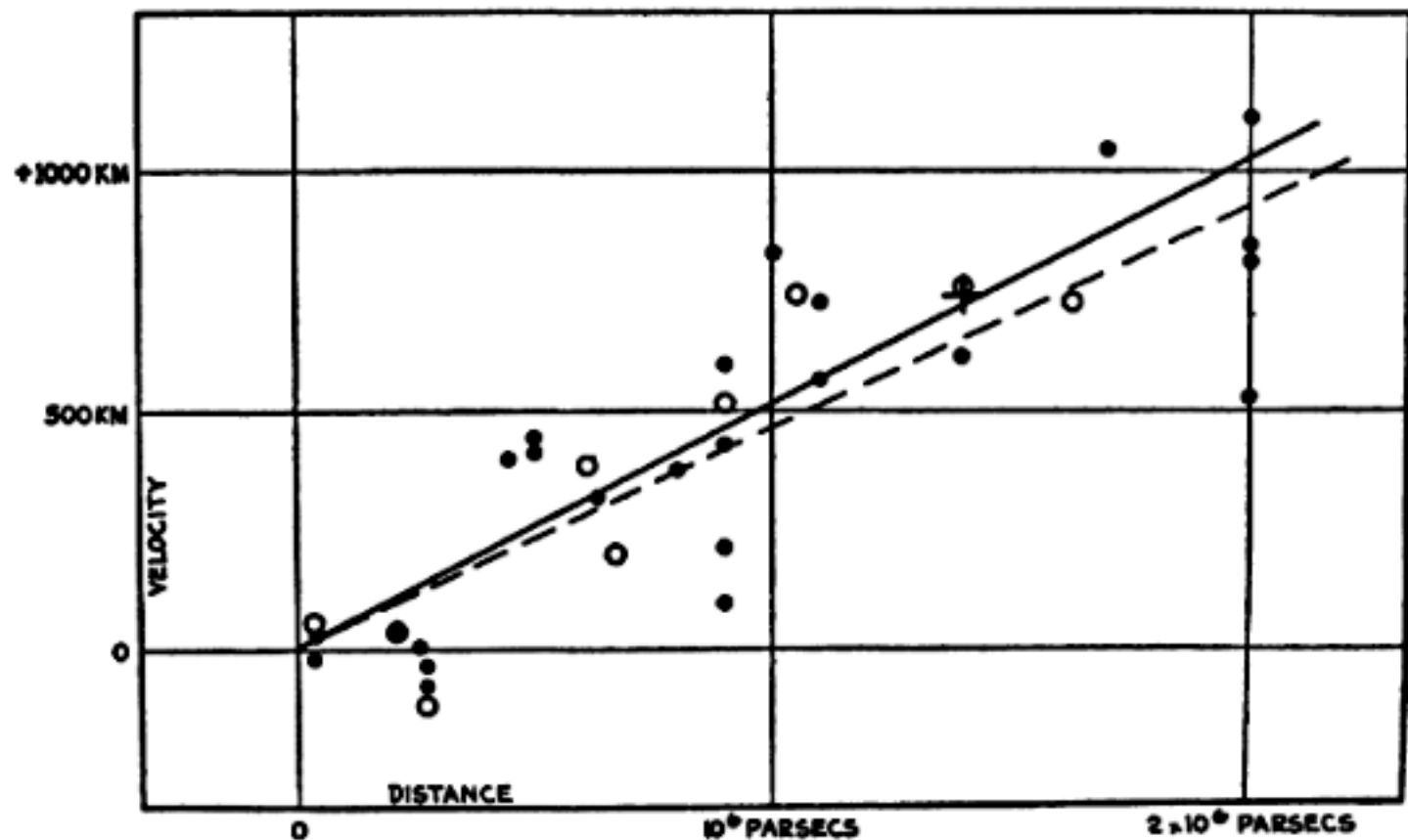


FIGURE 27.7 Hubble's 1936 velocity–distance relation. The two lines use different corrections for the Sun's motion. (Note: The vertical units should be km s^{-1} .) (Figure from Hubble, *Realm of the Nebulae*, Yale University Press, New Haven, CT, ©1936.)

Einstein's Biggest Blunder?

Einstein's initial formulation of General Relativity implied an **expanding Universe**.

The prevailing theory at the time, however, was that the Universe was static and infinite. Einstein included a constant (Λ) in his equations to make his theory predict a non expanding and static Universe.

Einstein later stated that the inclusion of this constant was the **greatest blunder of his life**.



Hubble showed with observations that the Universe was expanding. Later Einstein visited Hubble and congratulated him on this remarkable discovery.

Expanding Universe

The velocity of a galaxy through space is referred to as its **peculiar velocity**

The velocity of a galaxy due to expansion is referred to as **recession velocity**.

The motion of galaxies as they participate in the expansion of the Universe is called **Hubble flow**.

Example: Use Cosmocalc:

<http://www.astro.ucla.edu/~wright/CosmoCalc.html>

to determine the **comoving radial distance** at $z = 10$.

Expanding Universe

Example: Estimate the age of the Universe using the Hubble - Lemaitre Law

Expanding Universe

Example: *Estimate* the age of the Universe using the Hubble - Lemaitre Law

$$v = \frac{d}{t_{age}} \Rightarrow t_{age} = \frac{d}{v} = \frac{d}{H_0 d} = \frac{1}{H_0} = \frac{1}{70 \text{ km s}^{-1} \text{ Mpc}^{-1}} \Rightarrow$$
$$t_{age} \sim 14 \text{ billion years}$$

Estimating the Combined Mass of the Milky Way and Andromeda

The center of mass of N objects of mass m_i is:

$$\vec{R}_{cm} = \frac{\sum_1^N m_i \vec{r}_i}{M}$$

For binary system we can choose: $\vec{R}_{cm} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{M} = 0 \Rightarrow m_1 \vec{r}_1 + m_2 \vec{r}_2 = 0$ (1)

Define $\vec{r} = \vec{r}_2 - \vec{r}_1$. (2)

$$(1) \wedge (2) \Rightarrow \vec{r}_1 = \frac{-m_2 \vec{r}}{m_1 + m_2} = \frac{-\mu \vec{r}}{m_1} \quad \text{and} \quad \vec{r}_2 = \frac{m_1 \vec{r}}{m_1 + m_2} = \frac{\mu \vec{r}}{m_2}$$

Where we defined the reduced mass $\mu = \frac{m_1 m_2}{m_1 + m_2}$

The center of mass will not accelerate so if we define a reference frame centered on the center of mass it will be an inertial frame

Estimating the Combined Mass of the Milky Way and Andromeda

In the center-of-mass reference frame of the MW and Andromeda the total energy is:

$$E_{total} = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{|\vec{r}_2 - \vec{r}_1|}$$

$$E_{total} = \frac{1}{2}\mu v^2 - \frac{GM\mu}{r} \quad (3)$$

This is equivalent to considering a mass of $M = m_1 + m_2$ at the center of our coordinate system and the reduced mass μ located at a distance r . At the furthest separation ($2a$) between the MW and Andromeda the kinetic energy ~ 0 . The total energy was

$$E_{total} = -\frac{GM\mu}{2a} \quad (4)$$

$$(3) \wedge (4) \Rightarrow -\frac{GM\mu}{2a} = \frac{1}{2}\mu v^2 - \frac{GM\mu}{r} \Rightarrow v^2 = GM \left[\frac{2}{r} - \frac{1}{a} \right] \quad (5)$$

We can use Kepler's law for the semi-major axis: $P^2 = \frac{4\pi^2}{GM} a^3 \Rightarrow a = \left(\frac{4\pi^2}{P^2 GM} \right)^{-1/3} \quad (6)$

$$(3) \wedge (4) \Rightarrow v^2 = GM \left[\frac{2}{r} - \left(\frac{4\pi^2}{P^2 GM} \right)^{1/3} \right]$$

Estimating the Combined Mass of the Milky Way and Andromeda

$$v^2 = GM \left[\frac{2}{r} - \left(\frac{4\pi^2}{P^2 GM} \right)^{1/3} \right] \quad (5)$$

At the moment $r = 770$ kpc and $v = 119$ km/s

The period $P = t_H + t_{col}$ where t_H is the time since the big bang and t_{col} is the time from now to the collision $t_{col} = \frac{r}{v}$

$$P = t_H + t_{col} = t_H + \frac{r}{v}$$

Inserting r , v and P into (5) we obtain the total mass $M \sim 7.9 \times 10^{42}$ kg

Estimating the mass-to-light of Milkdromeda

$$M_{MW-A} \sim 7.9 \times 10^{42} \text{ kg} \sim 4 \times 10^{12} M_{\odot}$$

$$L_{MW} = 2.3 \times 10^{10} L_{\odot}$$

$$L_A = 4.6 \times 10^{10} L_{\odot}$$

$$\left(\frac{M}{L}\right)_{MW-A} = \left(\frac{4 \times 10^{12} M_{\odot}}{8.9 \times 10^{10} L_{\odot}}\right) = 58 \left(\frac{M_{\odot}}{L_{\odot}}\right)$$

Clusters of Galaxies

The Hercules cluster of galaxies just 650 million ly away.



Clusters of galaxies are the largest gravitationally bound objects in the Universe. Based on simulations the smallest structures collapsed first and eventually build the largest structures, such as clusters of galaxies (*bottom-up galaxy formation scenario*).

Properties of Clusters of Galaxies

Clusters of Galaxies contain between 100 - 2000 galaxies, hot X-ray emitting gas and dark matter. **Mass distribution:**

~5% in galaxies, ~10% in hot gas and ~85 % in dark matter.

- Total mass $\sim 10^{14} - 10^{15} M_{\odot}$
- Diameter of 1– 2 Mpc
- Velocity dispersion of $\sigma \sim 1000$ km/s
- $M/L \sim 500 M_{\odot}/L_{\odot}$

Clusters are classified as **rich** (> 100 members) **or poor** (< 100 members). Poor Clusters are also called **groups**.

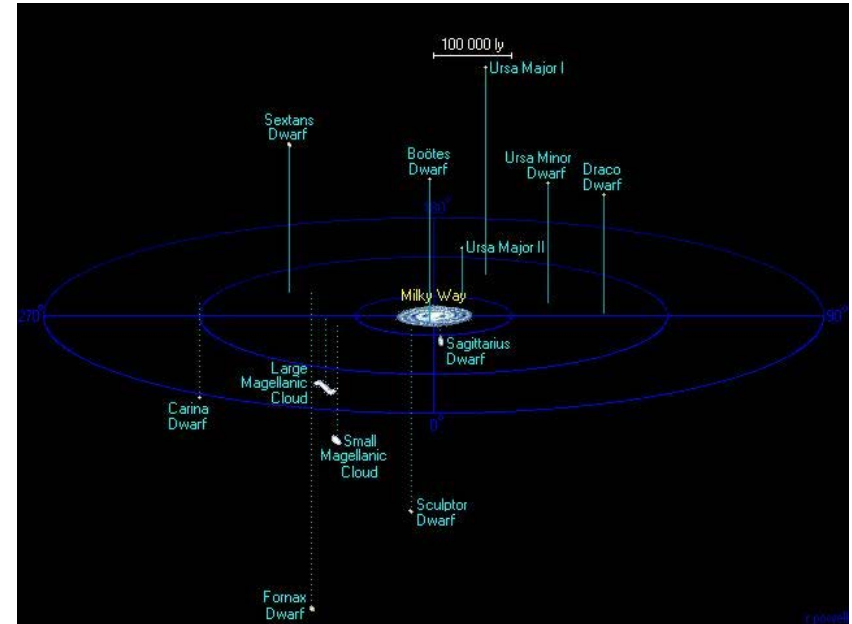
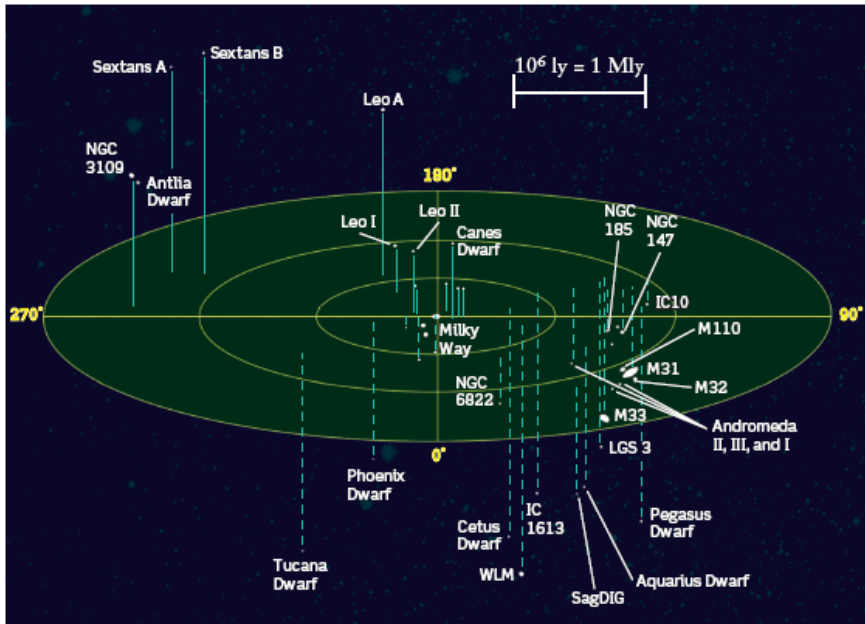
Cluster are also classified as **regular** (spherical shape) and **irregular** (non-spherical shape).

Properties of Groups

- Total mass $\sim 10^{13} M_{\odot}$
- Velocity dispersion of $\sigma \sim 150 \text{ km/s}$
- $M/L \sim 250 M_{\odot}/L_{\odot}$

The Milky Way belongs to the **Local Group**.

Local Group



Members of the **Local Group** include:

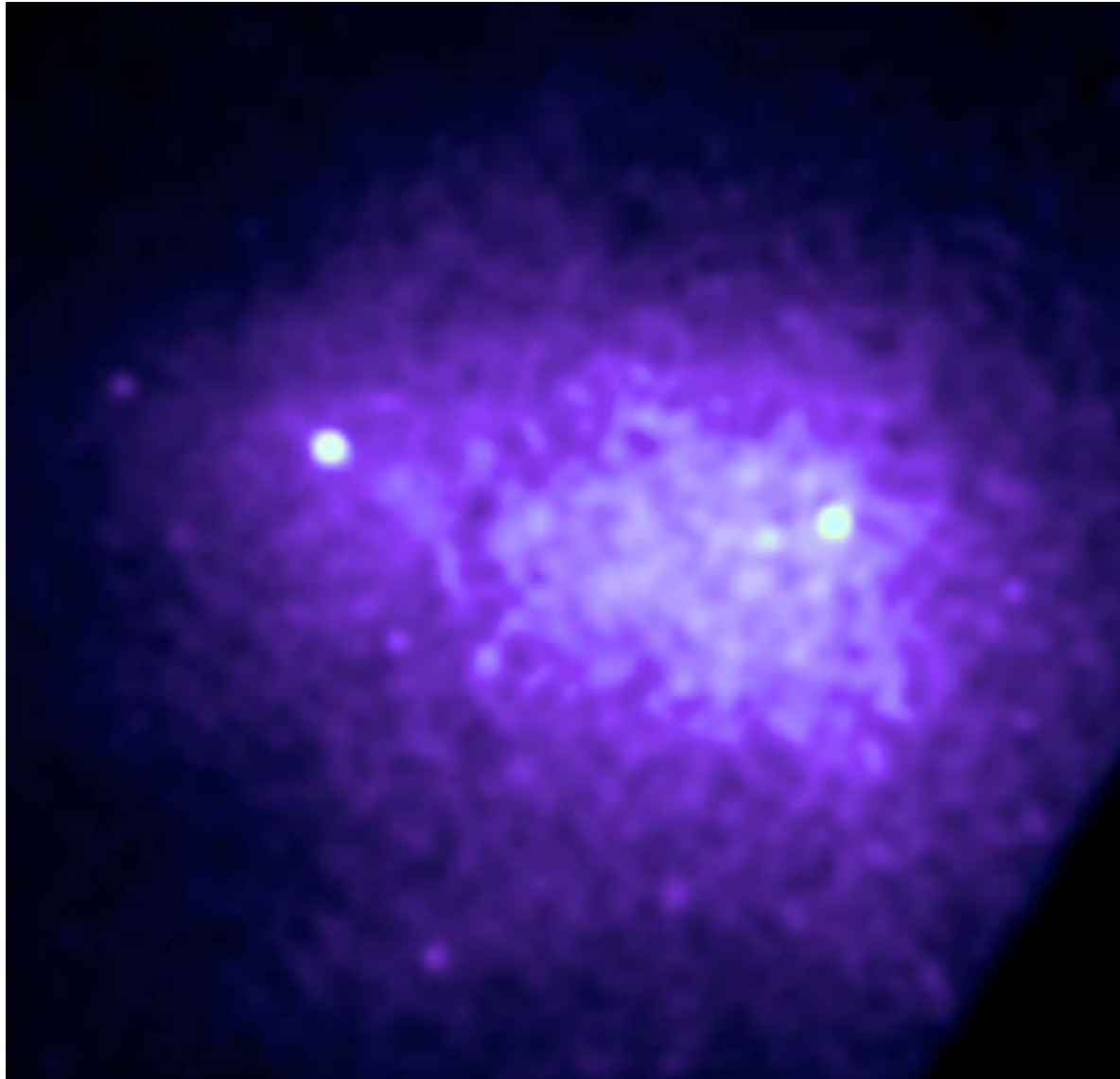
Milky Way, M31, M33, Large and Small Magellanic Clouds, Leo I and Leo II, Canis Dwarf (only 42,000 ly from the center of the Milky Way) Ursa minor and Major Dwarfs, Sextans Dwarf, Sagittarius Dwarf.

Virgo: A Rich Irregular Cluster of Galaxies

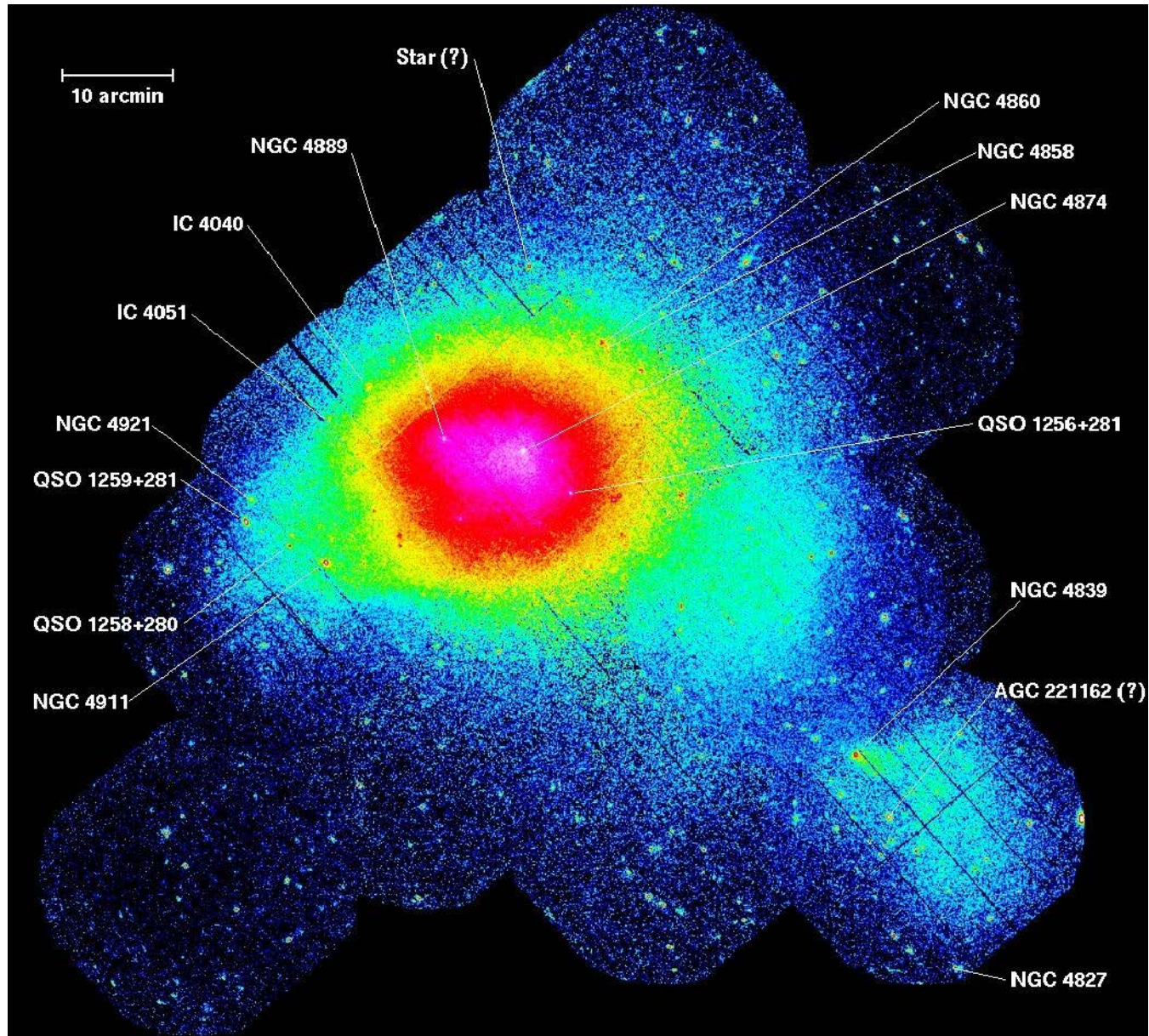


FIGURE 27.13 The center of the Virgo cluster, showing the giant ellipticals M84 (right) and M86 (center). (Courtesy of National Optical Astronomy Observatories.)

Coma: A Rich Regular Cluster of Galaxies



Coma: A Rich Regular Cluster of Galaxies



Coma Cluster of galaxies

Hot Gas In Clusters of Galaxies

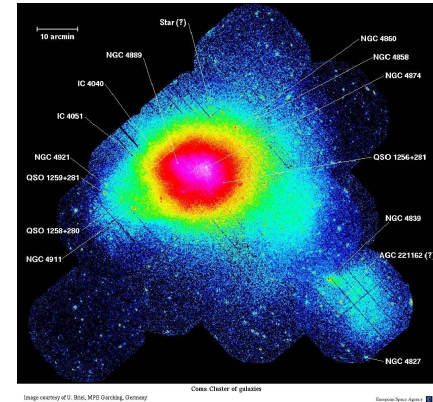
Most galaxy clusters contain **hot gas primarily heated during the cluster formation/collapse**. The hot gas emits *Bremsstrahlung* radiation .

One can use the observed temperature and density profiles of GCs to infer the total mass of a GC.

$$dm \frac{d^2 r}{dt^2} = - \frac{GMdm}{r^2} - dPA \quad (1)$$

$$dm = \rho dV = \rho A dr \quad (2)$$

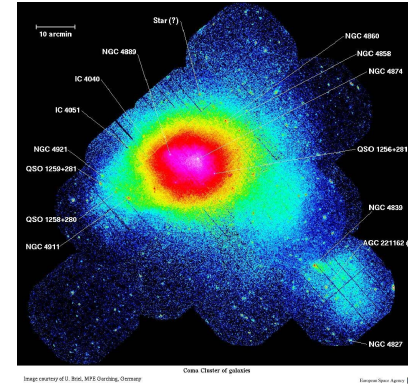
$$(1) \wedge (2) \Rightarrow \frac{dP}{dr} = - \frac{GM\rho}{r^2} \Rightarrow M = - \frac{r^2}{G\rho} \frac{dP}{dr} \quad (3)$$



Hot Gas In Clusters of Galaxies

$$\frac{dP}{dr} = \frac{dnkT}{dr} = kT \frac{dn}{dr} + kn \frac{dT}{dr} \quad (4)$$

$$\rho = \frac{M}{V} = \frac{N\mu m_H}{V} = n\mu m_H \quad (5)$$



$$(3) \wedge (4) \wedge (5) \Rightarrow M(< r) = -\frac{r^2}{G\rho} \frac{dP}{dr} = -\frac{r^2}{Gn\mu m_H} \left[kT \frac{dn}{dr} + kn \frac{dT}{dr} \right]$$

$$\Rightarrow M(< r) = -\frac{r^2 kT}{G\mu m_H} \left[\frac{1}{n} \frac{dn}{dr} + \frac{1}{T} \frac{dT}{dr} \right]$$

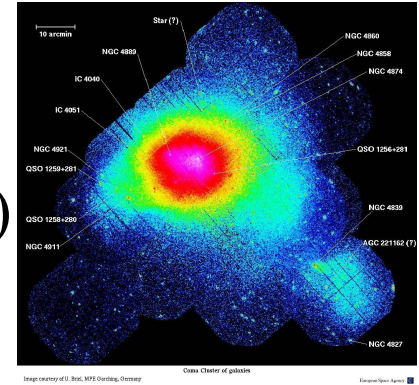
$M(< r)$ is the total mass within a radius r (dark matter + gas + stars +...)

Mass-to-Light of M87 and Coma

1. M87

$$M_{87}(r < 300 \text{ kpc}) \sim 3 \times 10^{13} M_{\odot}$$

$$\left(\frac{M}{L}\right)_{87} \simeq 750 \left(\frac{M_{\odot}}{L_{\odot}}\right) \text{ (M includes stars, gas and dark matter)}$$



2. Coma

Zwicky figured out that clusters contained a large fraction of Dark Matter by using the virial theorem

$$\sigma_r = 977 \frac{\text{km}}{\text{s}}, R = 3 \text{ Mpc}, L = 5 \times 10^{12} L_{\odot}$$

$$M_{\text{virial}} = \frac{5\sigma_r^2 R}{G} = \frac{5 \left(977 \times 10^3 \frac{\text{km}}{\text{s}}\right)^2 (3 \times 3.086 \times 10^{22} \text{m})}{6.67 \times 10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}}$$

$$\left(\frac{M}{L}\right)_{\text{Coma}} = \frac{3.3 \times 10^{15}}{5 \times 10^{12}} \left(\frac{M_{\odot}}{L_{\odot}}\right) \simeq 662 \left(\frac{M_{\odot}}{L_{\odot}}\right)$$

Thermal Bremsstrahlung (free-free emission)

Energy emitted per unit time per unit volume between frequencies ν and $\nu + d\nu$

$$l_\nu d\nu = 5.44 \times 10^{-52} (4\pi n_e^2) T^{-1/2} e^{\frac{-h\nu}{kT}} d\nu \frac{W}{m^3}$$

Where n_e is the electron density and T is the temperature

Luminosity density (over all frequencies)

$$L_{Volume} = 1.42 \times 10^{-40} (n_e^2) T^{1/2} \frac{W}{m^3}$$

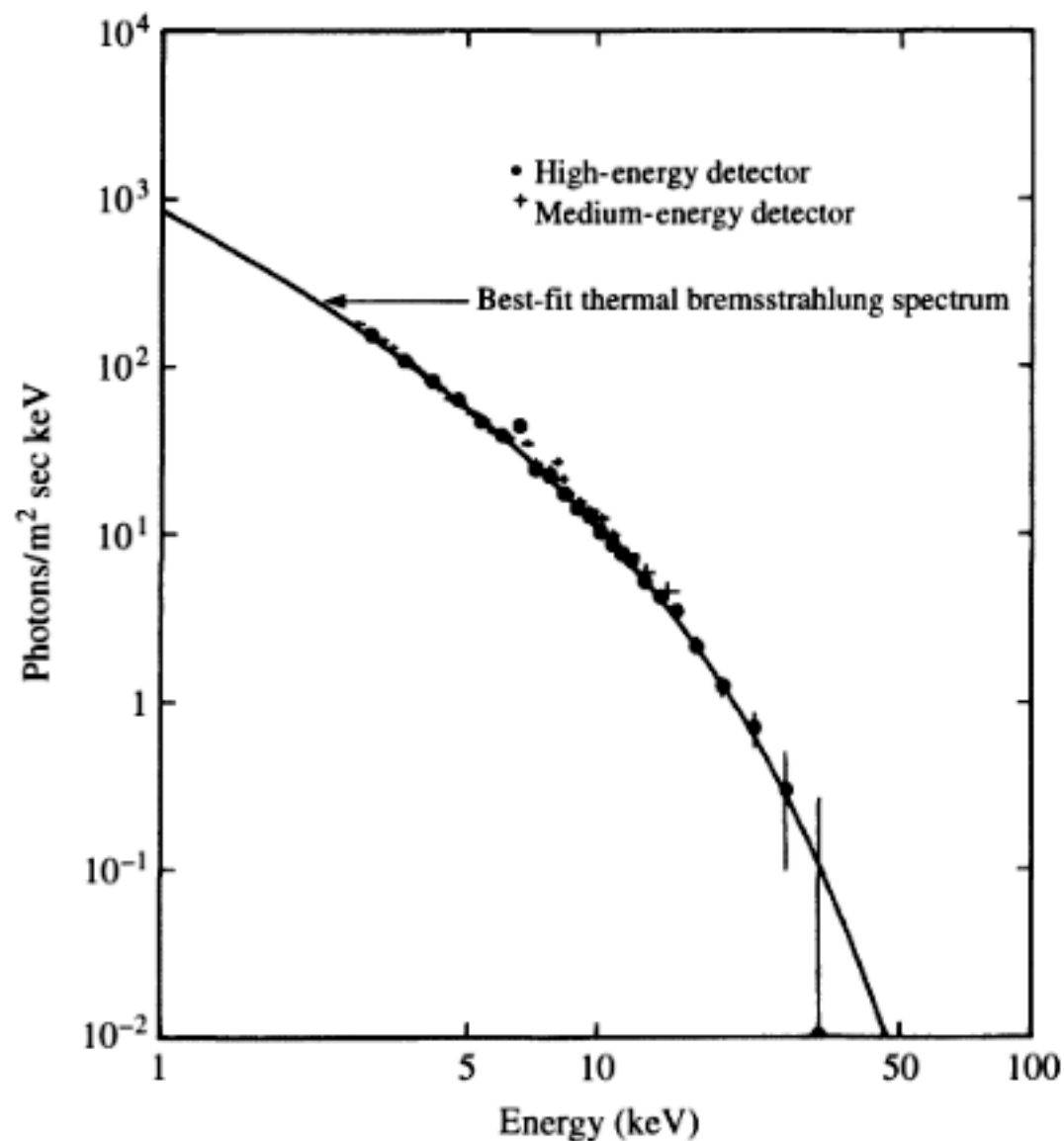


FIGURE 27.17 Thermal bremsstrahlung spectrum (line) for 88 million K. The points are observations of X-rays from the Coma cluster's intracluster gas. Photon energy is plotted on the horizontal axis. (Figure adapted from Henriksen and Mushotzky, *Ap. J.*, 302, 287, 1986.)

Estimate of the Mass of the Hot Gas in the Coma cluster

Assumptions:

(1) Coma gas is modeled as **an isothermal sphere** of hot ionized hydrogen.

(2) $R \sim 1.5$ Mpc

(3) **Gas is optically thin**, meaning the probability that photons from the source are absorbed or scattered is low and all photons emitted along the line of sight are observed.

The temperature and luminosity are determined by modeling the observed X-ray spectrum: $T \sim 8.8 \times 10^7 \text{ K}$ and $L_X \sim 5 \times 10^{37} \text{ W}$

Estimate of the Mass of the Hot Gas in the Coma cluster

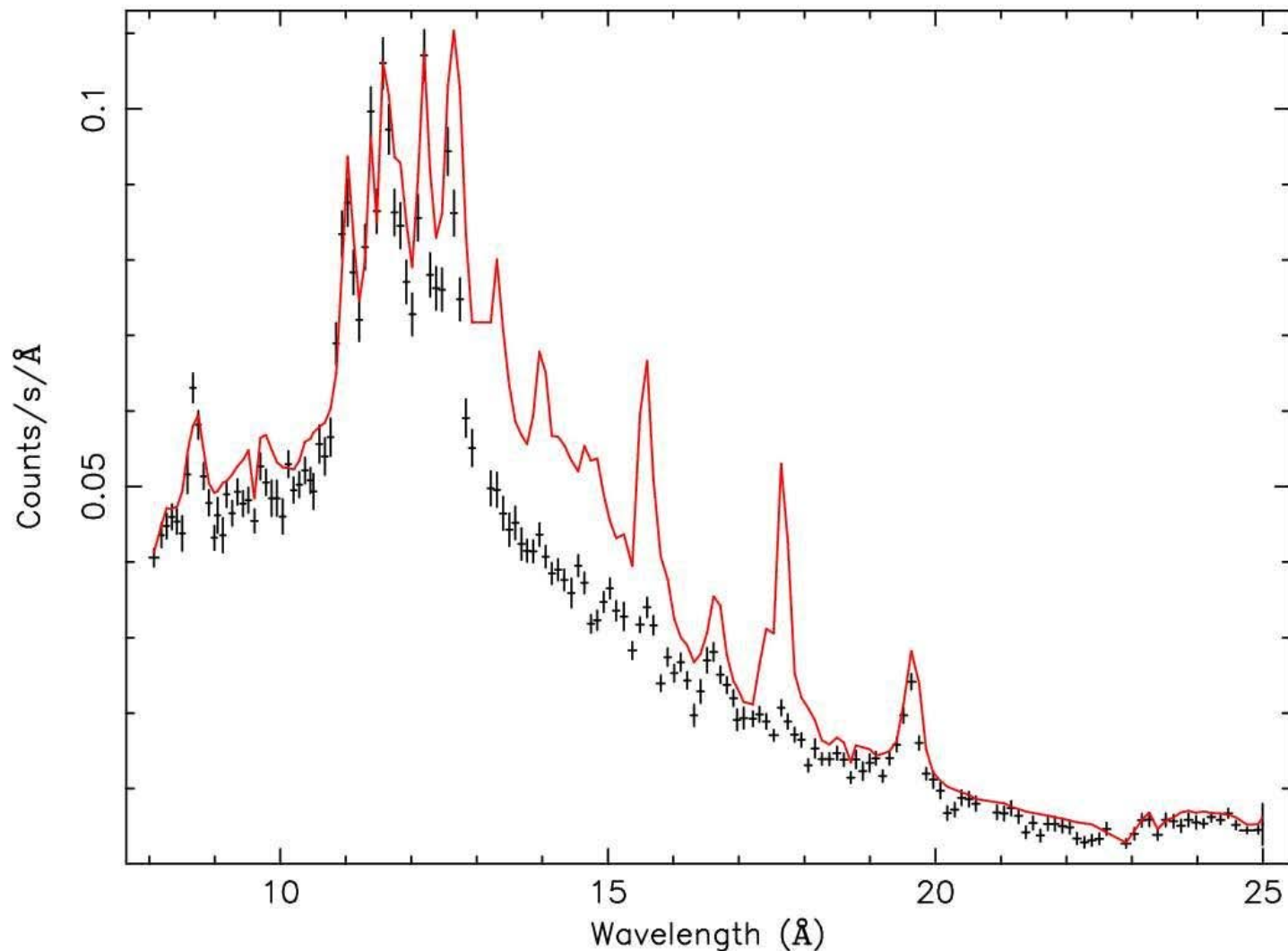
The temperature and luminosity are determined by modeling the observed X-ray spectrum: $T \sim 8.8 \times 10^7 \text{ K}$ and $L_X \sim 5 \times 10^{37} \text{ W}$

$$L_X = L_{Vol} \times \text{Volume} = L_{Vol} \frac{4}{3} \pi R^3 \Rightarrow$$

$$L_X = 1.42 \times 10^{-40} (n_e^2) T^{1/2} \frac{4}{3} \pi R^3 \Rightarrow n_e = \left(\frac{L_X}{1.42 \times 10^{-40} T^{1/2} \frac{4}{3} \pi R^3} \right)^{1/2}$$

n_e is the number density of e^- . For completely ionized H there is 1 H atom per every free e^- : $n_e = n_H$

$$\rho = \frac{M}{V} \Rightarrow M_{gas} = \rho V = n_H m_H V = n_e m_H V = n_e m_H \frac{4}{3} \pi R^3$$



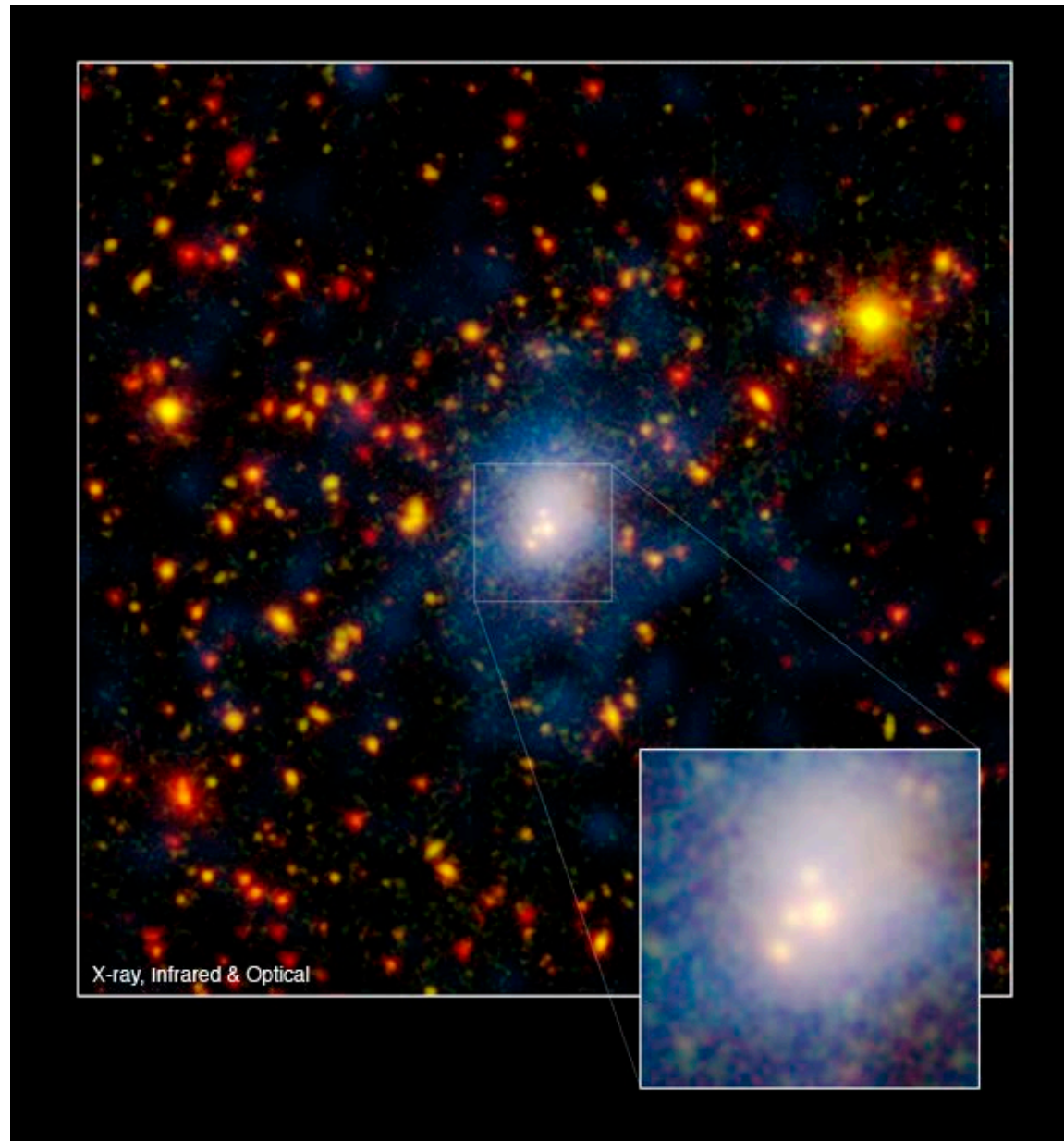
Slow cooling in the core of the galaxy cluster 2A 0335+096

Image courtesy of Jelle de Plaa, SRON, NL.

European Space Agency 

XMM-Newton spectrum of the cluster of galaxies 2A 0335+096. The black crosses are the data points, the red line shows the predicted spectrum according to the previously favored isobaric cooling-flow model. The difference between the two, due to the lack of the expected Fe XVII lines, indicates that the cores of clusters are cooling much slower than predicted.

$z=0.39$ CL0958+4702: Spitzer and Chandra Spy Monster Galaxy Pileup

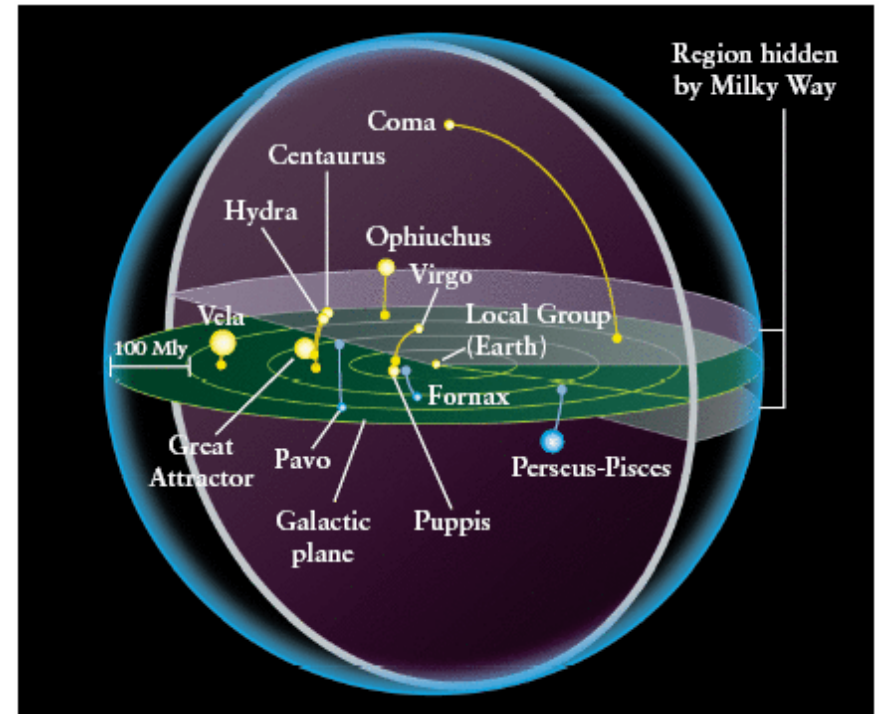


Superclusters

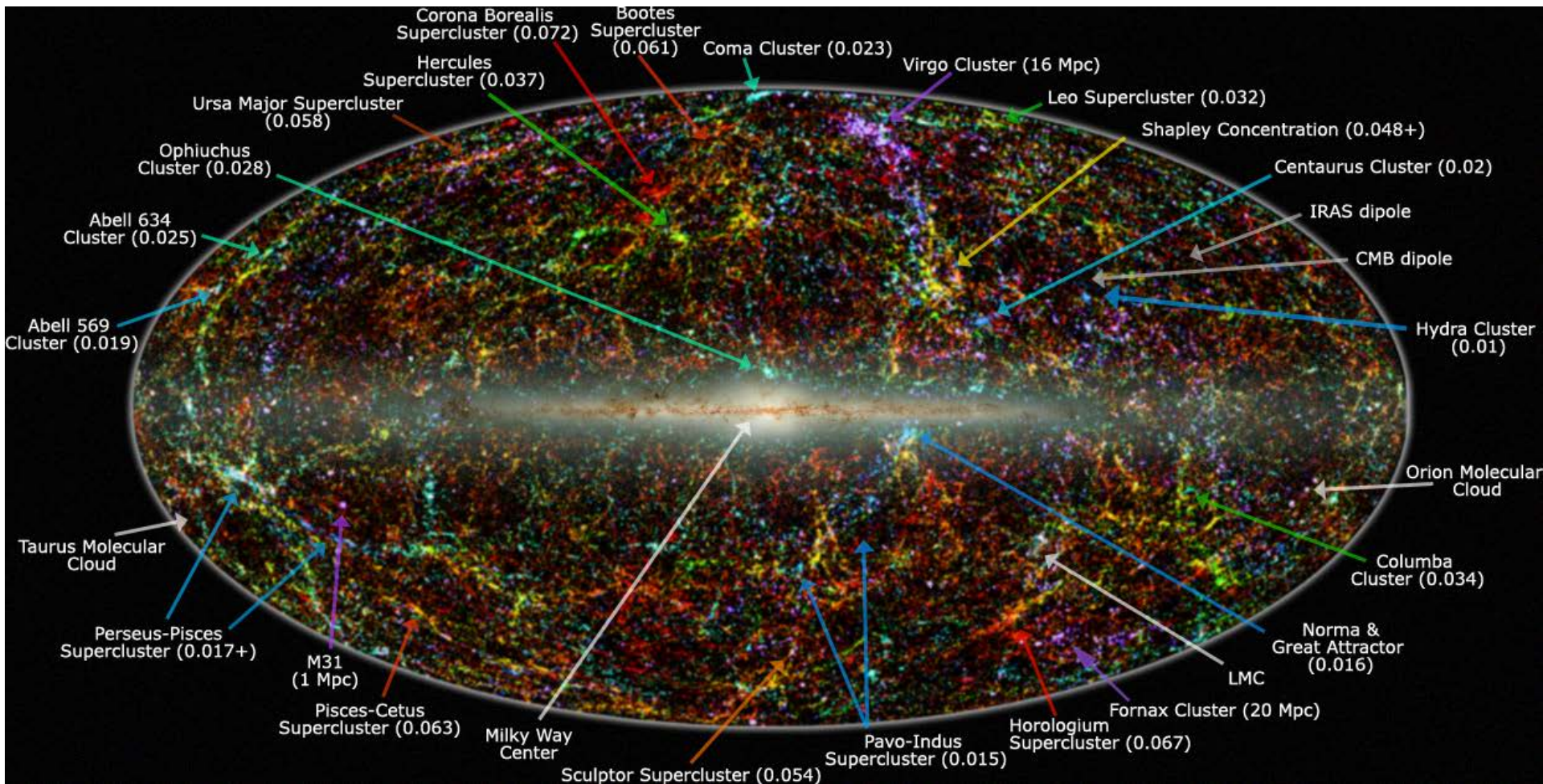
Clusters of galaxies are also grouped in larger structures called **superclusters**.

A supercluster may contain tens of clusters of galaxies.

Superclusters are not bound together by gravity and the distances between members grows as the Universe expands.



Nearby clusters of galaxies



Panoramic view of the entire near-infrared sky. The location of the Great Attractor is shown following the long blue arrow at bottom-right.

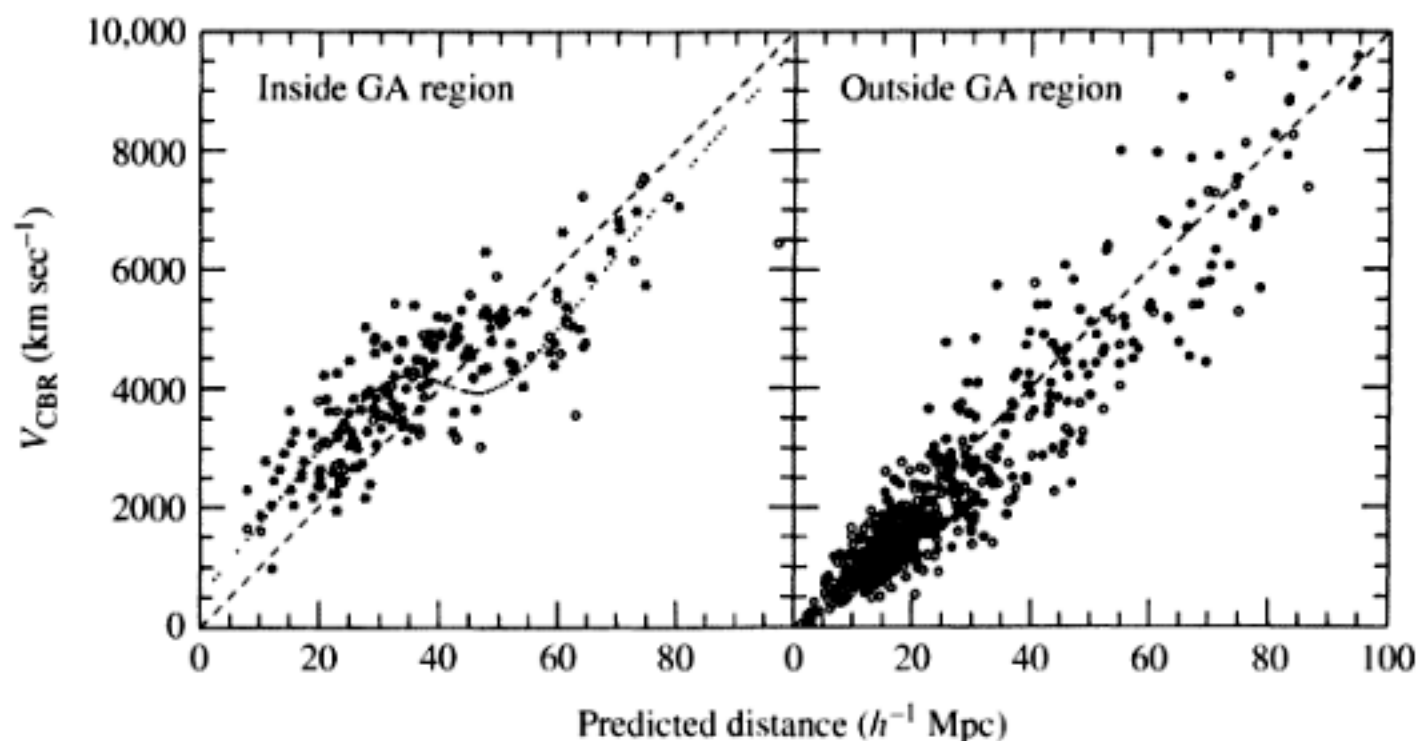


FIGURE 27.19 Left: velocities of galaxies in the Centaurus region, compared with the Hubble flow (dashed line). The dotted line shows the theoretical variation in velocity produced by a model of the Great Attractor. The Hydra–Centaurus supercluster is centered at about $30h^{-1}$ Mpc. Right: comparison figure for galaxies observed in another direction. (Figure adapted from Dressler and Faber, *Ap. J. Lett.*, 354, L45, 1990.)

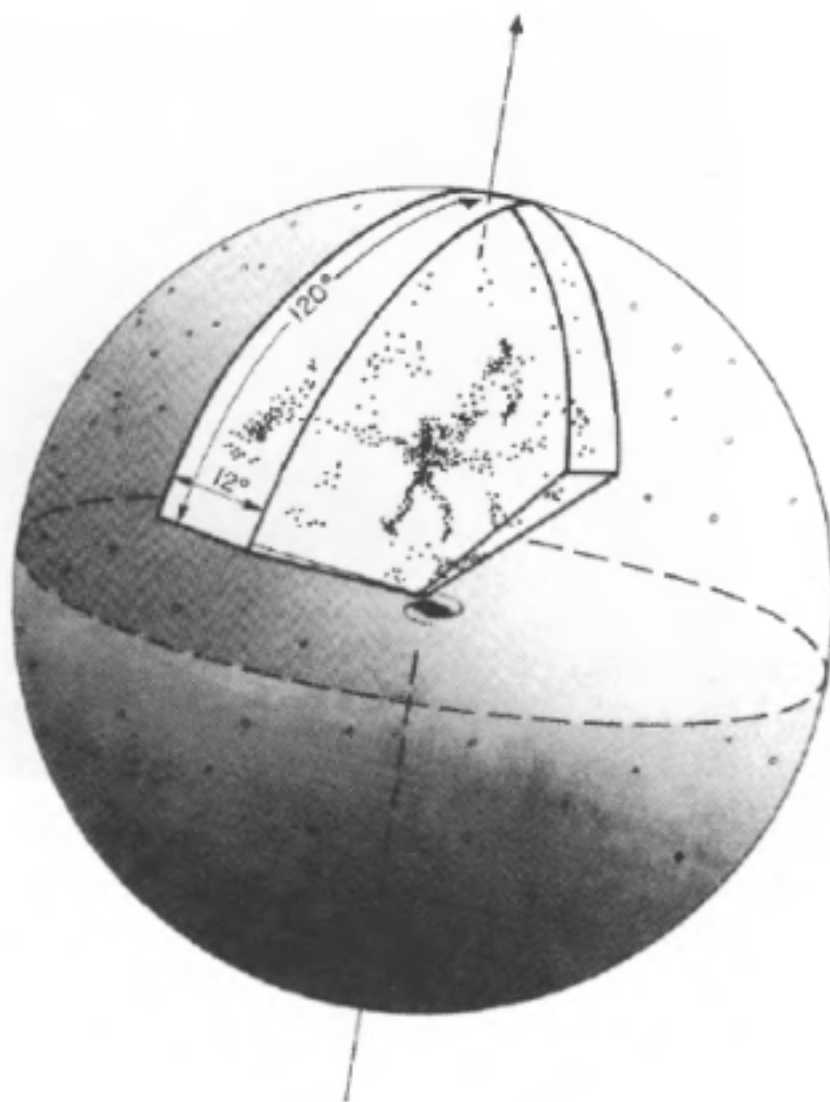
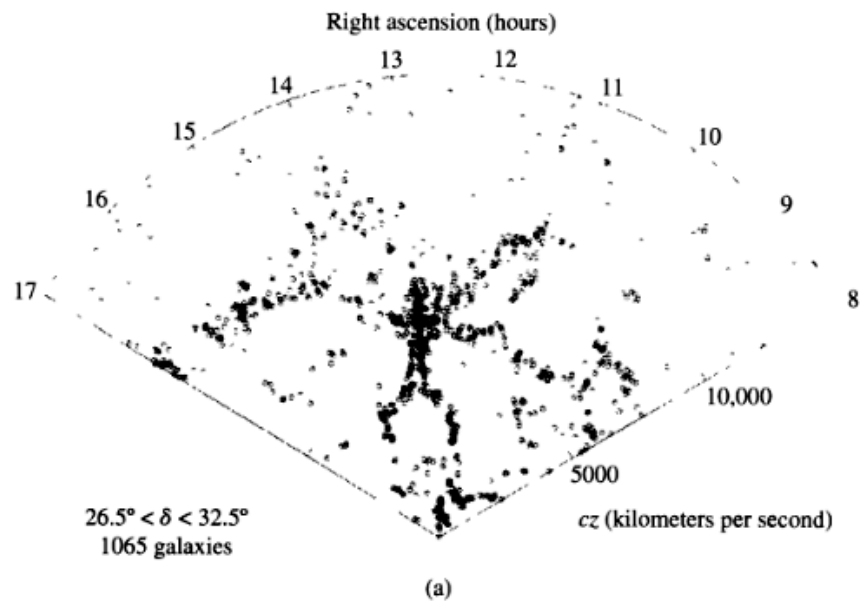
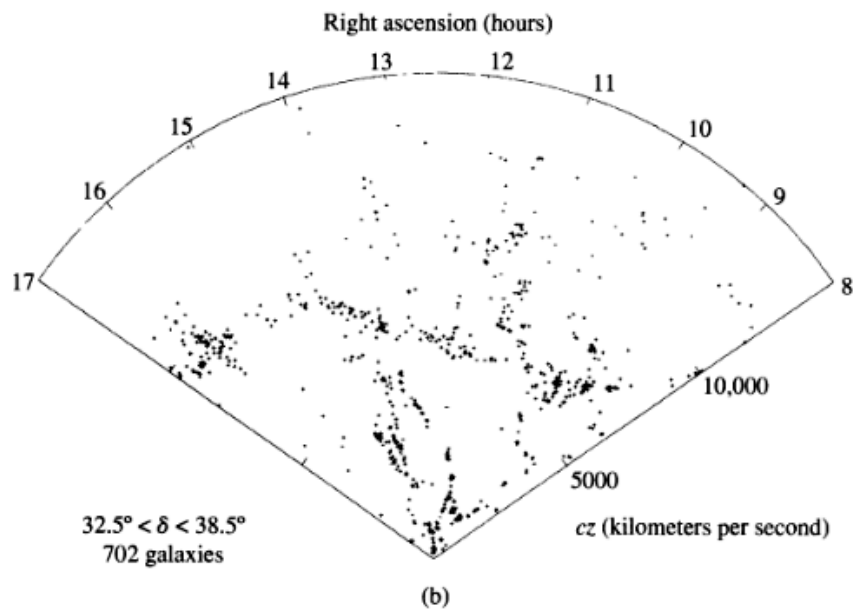


FIGURE 27.22 Two adjacent 6° -wide wedges used in the CfA redshift survey. (Figure from Geller, *Mercury*, May/June 1990.)



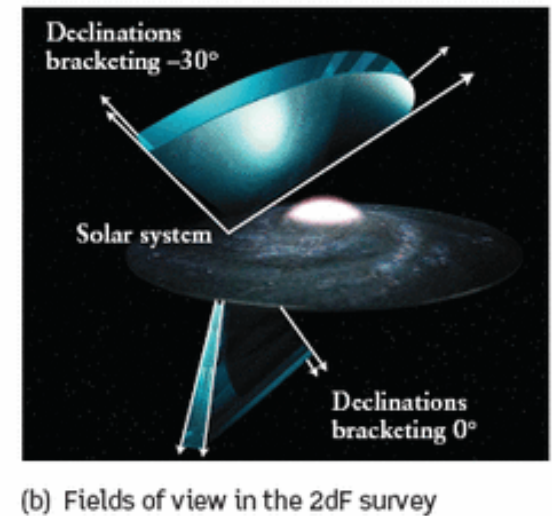
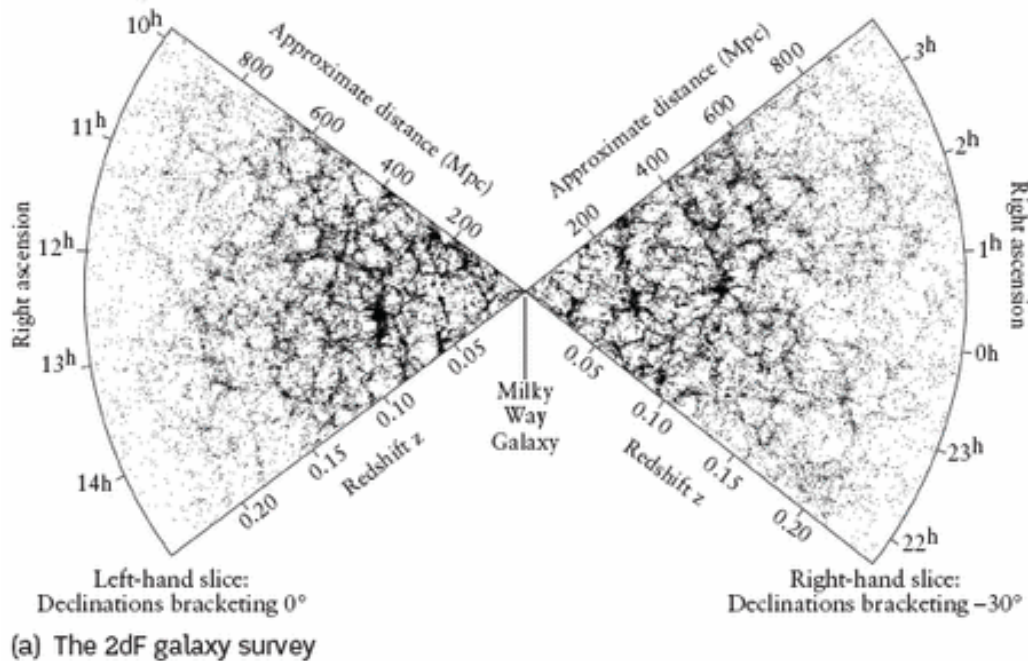
(a)



(b)

FIGURE 27.23 Two slices from the CfA redshift survey. Right ascension is indicated along the top of each figure. (a) $26.5^\circ < \delta < 32.5^\circ$. (b) $32.5^\circ < \delta < 38.5^\circ$. (Figures adapted from Geller, *Mercury*, May/June 1990.)

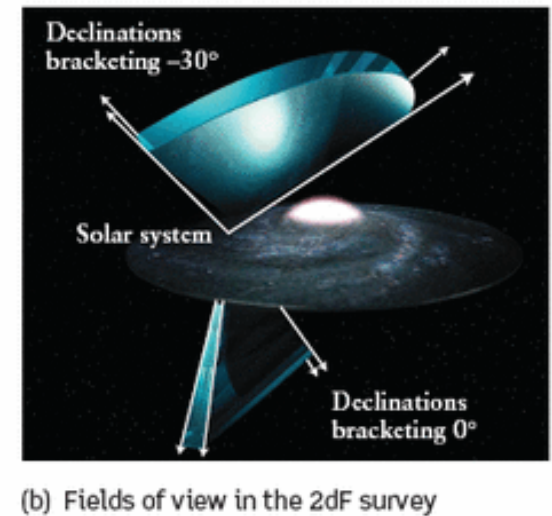
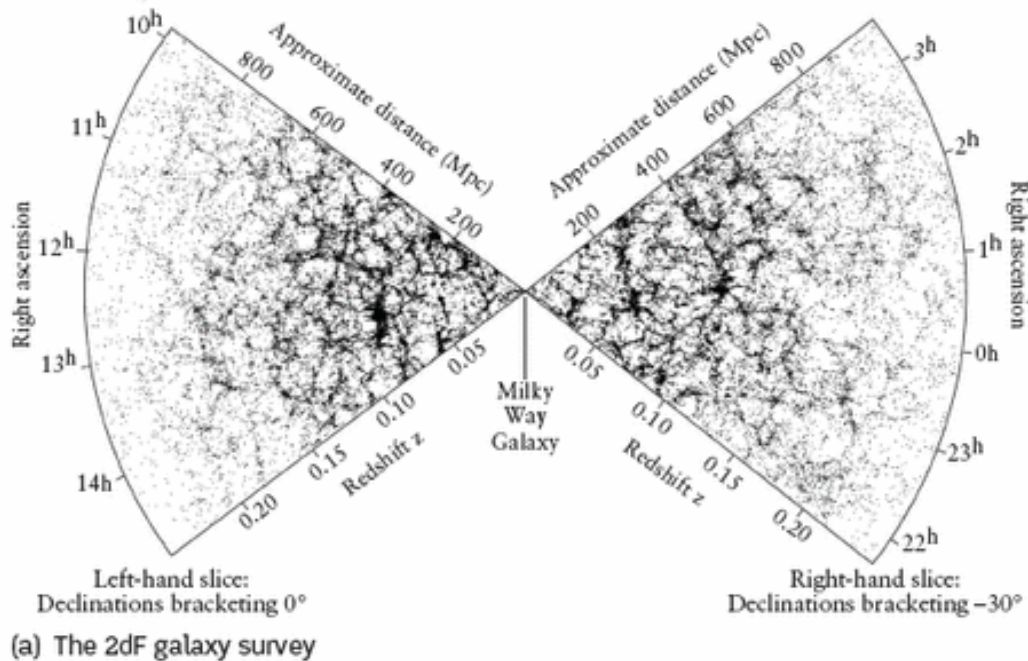
Large Scale Distribution of Galaxies: Voids and Sheets



Three dimensional maps of galaxies in the nearby Universe are now available from the **Sloan Digital Sky Survey (SDSS)** and the **two degree field galactic redshift survey (2dFGRS)**.

These maps indicate that most galaxies are located on sheets several Mpc thick and separated by voids of sizes ~ 100 Mpc.

Large Scale Distribution of Galaxies: Voids and Sheets



This pattern is similar to that of suds in a kitchen sink, with sheets of soap film (analogous to galaxies) surrounding air bubbles (analogous to voids). Voids contain very few galaxies but may contain hydrogen clouds.

On scales much larger than 100 Mpc, the distribution of galaxies in the universe appears to be roughly uniform.

Clustering of Galaxies

Assume that galaxies uniformly occupy a volume dV with an average number density of $n = \frac{dN}{dV}$

The probability of finding a galaxy in a volume dV is:

$$dP \approx n dV \text{ (for } dV \text{ small enough } dP \ll 1)$$

Galaxies however are not randomly distributed in space and the number density is not constant at all scales.

The probability of finding a galaxy in a volume dV at a distance r from another galaxy is:

$$dP \approx n [1 + \xi(r)] dV$$

where $\xi(r)$ is the two-point correlation function that describes whether galaxies are more concentrated or less than average.

Clustering of Galaxies

From a study of the clustering properties of a sample of $\sim 120,000$ SDSS galaxies and for separations of $0.1 h^{-1} \text{ Mpc} < r < 16 h^{-1} \text{ Mpc}$ the correlation function is found to be:

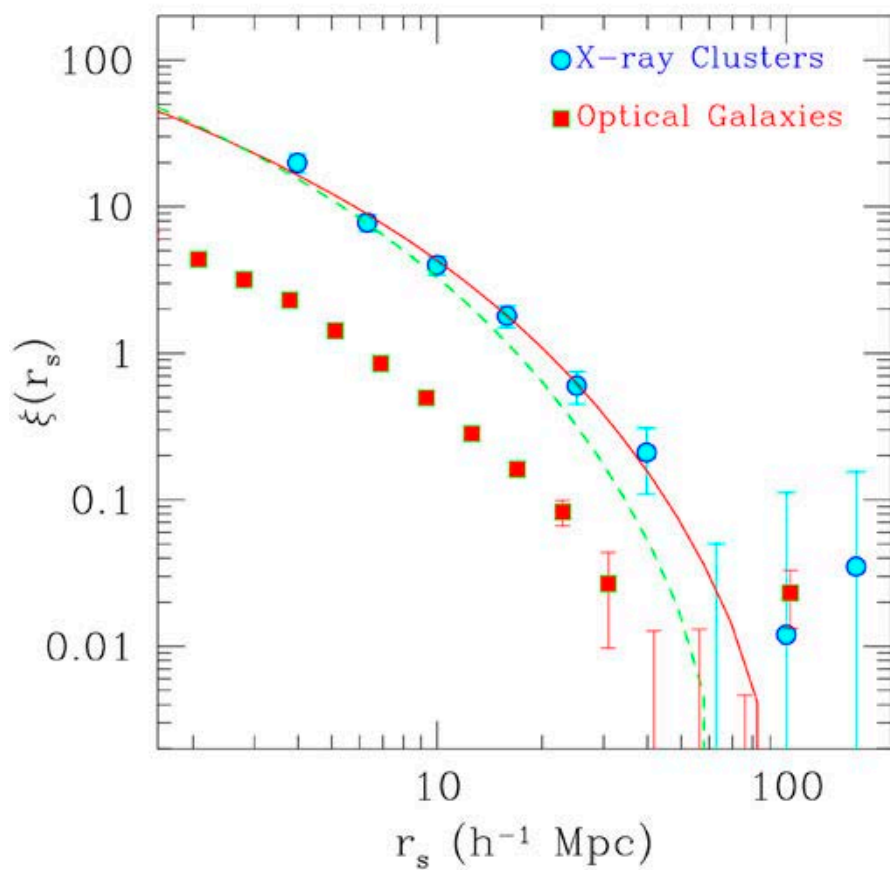
$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$

Where $r_0 = 5.53 h^{-1} \text{ Mpc}$, $\gamma = 1.8$ and $H_0 = h \text{ } 100 \text{ km s}^{-1} \text{ Mpc}^{-1}$

The correlation length r_0 varies with galaxy luminosity from $4.7 h^{-1} \text{ Mpc} - 7.4 h^{-1} \text{ Mpc}$

Longer correlation lengths for more luminous galaxies

Clustering of Galaxies



$$\xi(r) = \left(\frac{r}{r_0}\right)^{-\gamma}$$

The two-point correlation function for the X-ray selected clusters from the REFLEX survey (circles) and for the Las Campanas galaxy redshift survey (squares). The solid and dashed lines are the expected results for an X-ray similar survey in a LCDM model with different values for the cosmological parameters.

https://ned.ipac.caltech.edu/level5/March04/Jones/Jones5_3.html