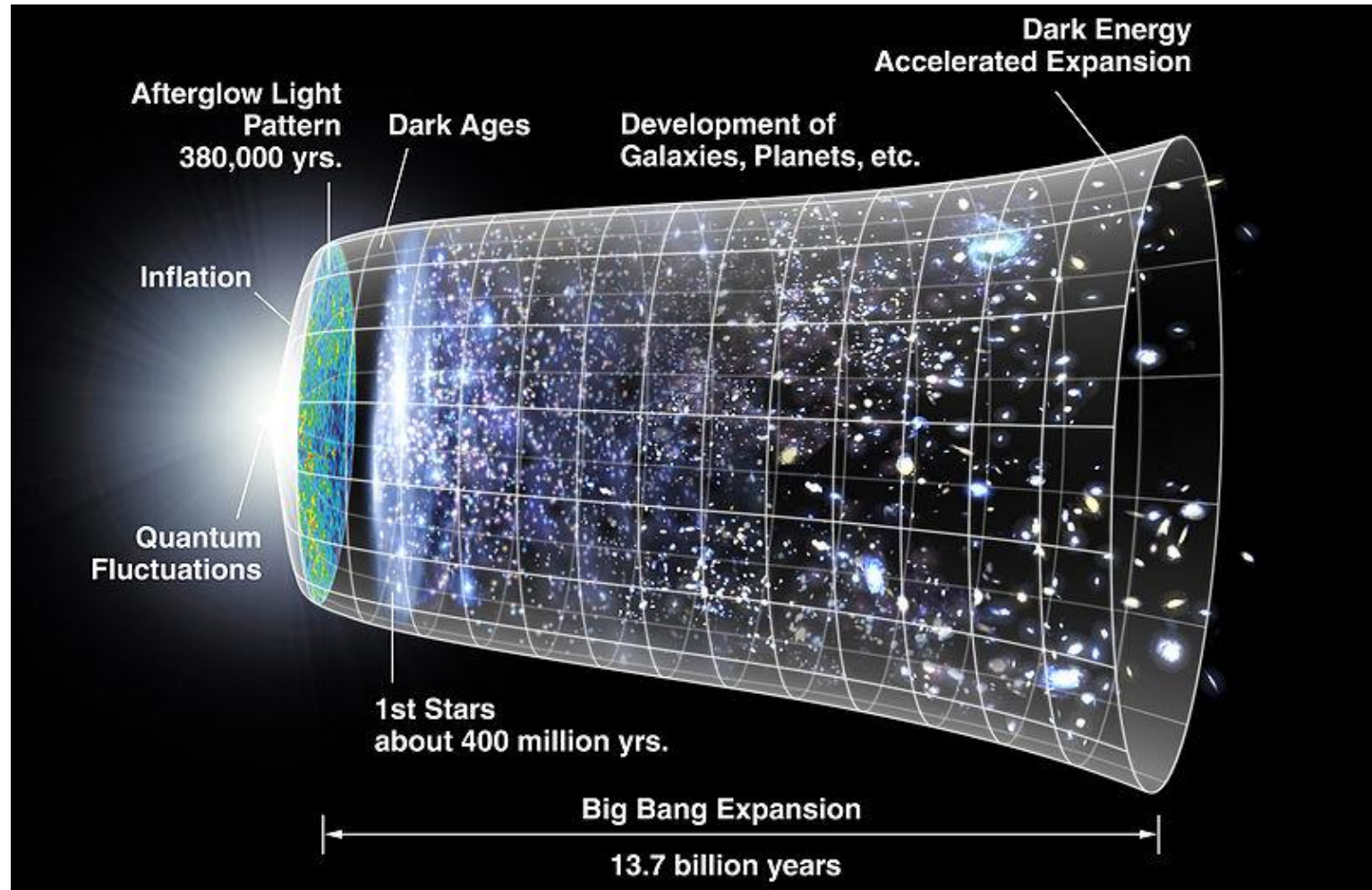


Cosmology



Cosmological Principle

Cosmological Principle: Observers on Earth do not occupy any special location in the Universe. Two consequences of the cosmological principle are homogeneity and isotropy.

Homogeneous Universe:

The same observational evidence is available from any part of the Universe. It doesn't matter where you are located in the Universe.

Isotropic:

Over sufficiently large distances the Universe looks the same in any direction we look.

Cosmological Redshift

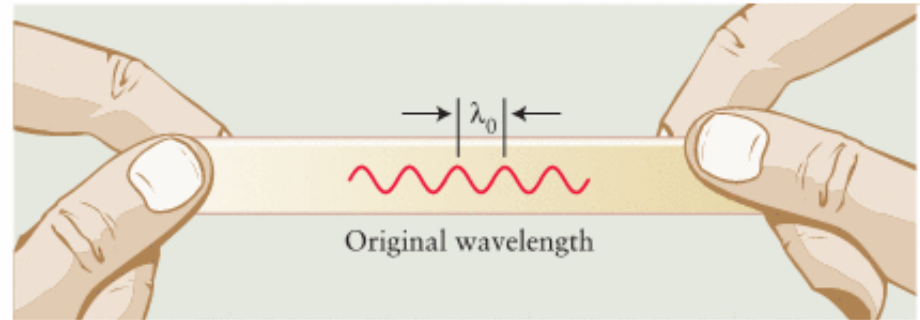
A redshift caused by the expansion of the universe is called cosmological redshift.

We can easily calculate the factor by which the Universe has expanded from some previous time as follows:

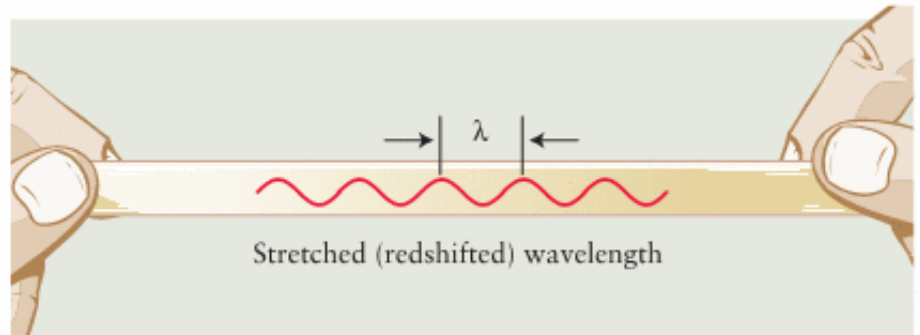
$$z = (\lambda_{\text{obs}} - \lambda_0) / \lambda_0 \rightarrow \lambda_{\text{obs}} / \lambda_0 = (1+z)$$

This means that if you observe an object to have a redshift of $z = 1$ the distance between us and the object has increased by a factor of 2 from the time the photon left that object and arrived to Earth.

How does the volume and density change?



(a) A wave drawn on a rubber band ...



(b) ... increases in wavelength as the rubber band is stretched.

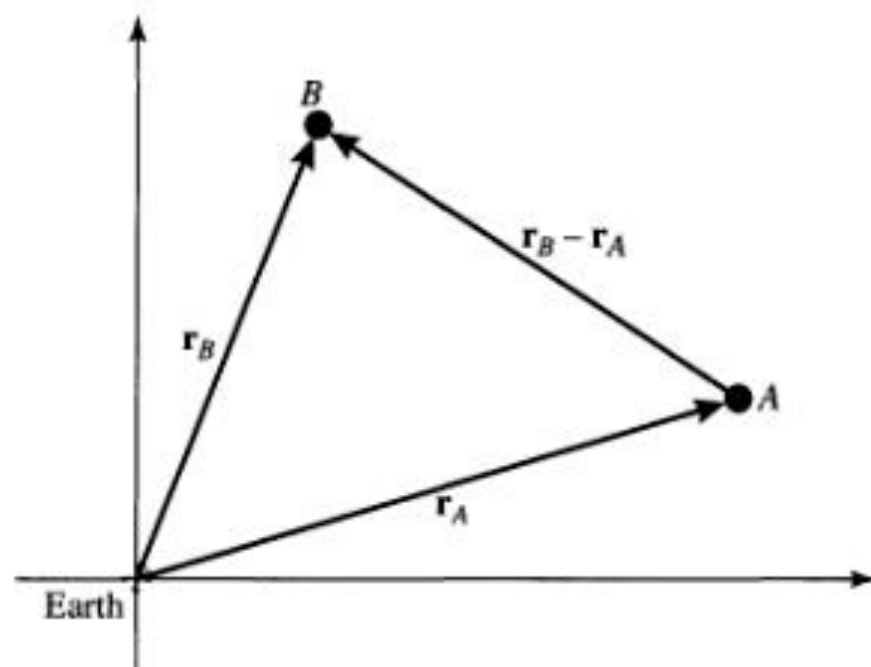


FIGURE 29.1 The expansion of the universe, with Earth at the origin.

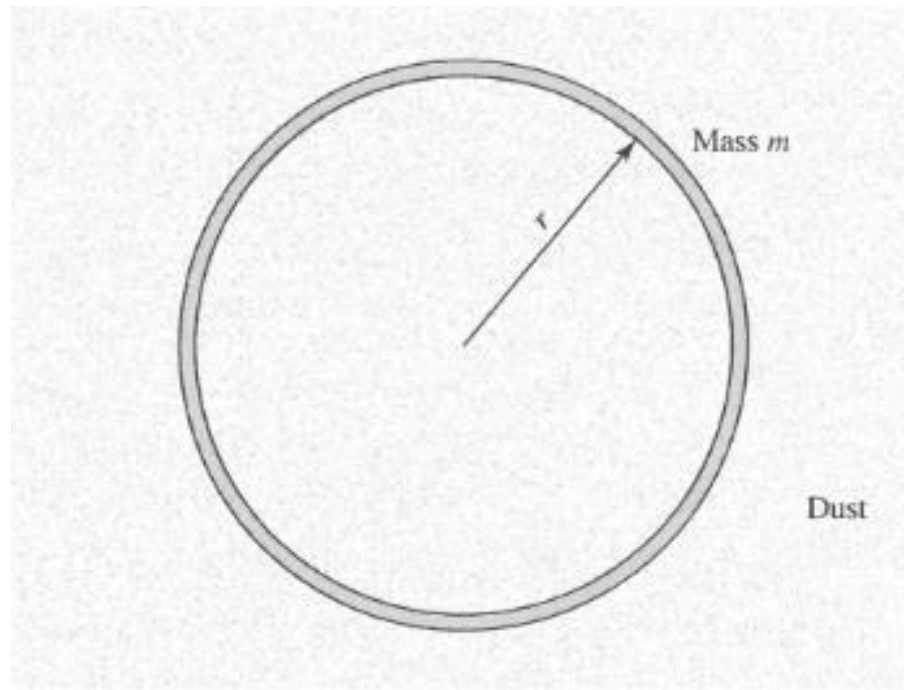


FIGURE 29.2 Spherical mass shell in a dust-filled universe.

The expansion of the Universe is represented by a scale factor $R(t)$: $r(t) = R(t)r(t_0)$

By definition $R(t_0) = 1$

Relation between scale factor and redshift: $R = \frac{1}{1+z}$

Curvature of the Universe

The **Density Parameter of the Universe Ω_0** is defined as the ratio of the combined mass density ρ_0 to the critical mass density ρ_c :

$$\Omega_0 = \rho_0 / \rho_c$$

$$\rho_0 = \rho_m + \rho_{rel} + \rho_\Lambda$$

Closed Universe: $\rho_0 > \rho_c \rightarrow \Omega_0 > 1$

Flat Universe: $\rho_0 = \rho_c \rightarrow \Omega_0 = 1$

Open Universe: $\rho_0 < \rho_c \rightarrow \Omega_0 < 1$

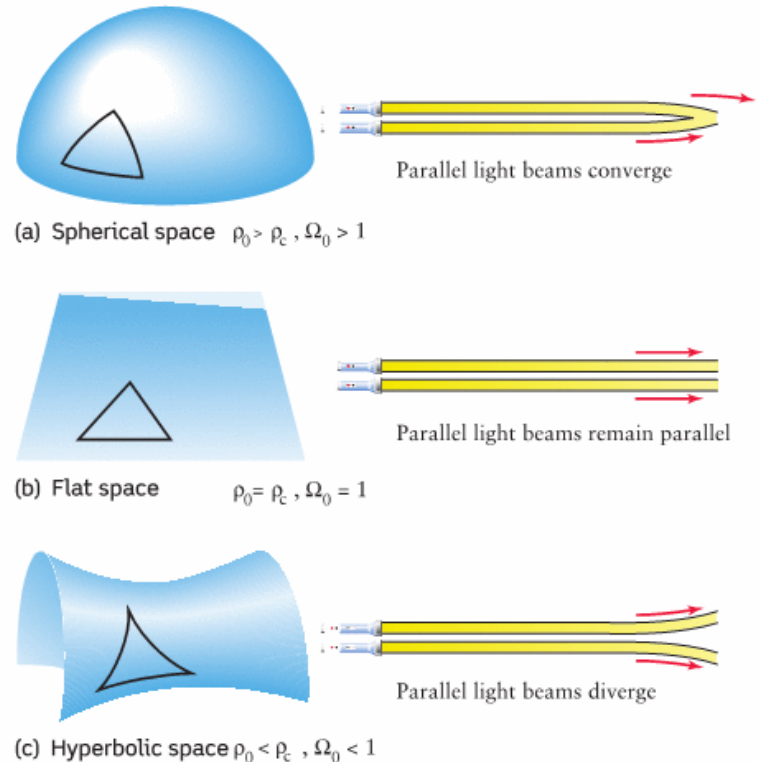
Where the critical mass density is:

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

Where the Hubble constant H_0 is the rate of expansion of the Universe.

Cepheid/SN method yield $H_0 = 70 - 74$ km/s/Mpc

CMB (Planck) yield $H_0 = 67 - 68$ km/s/Mpc



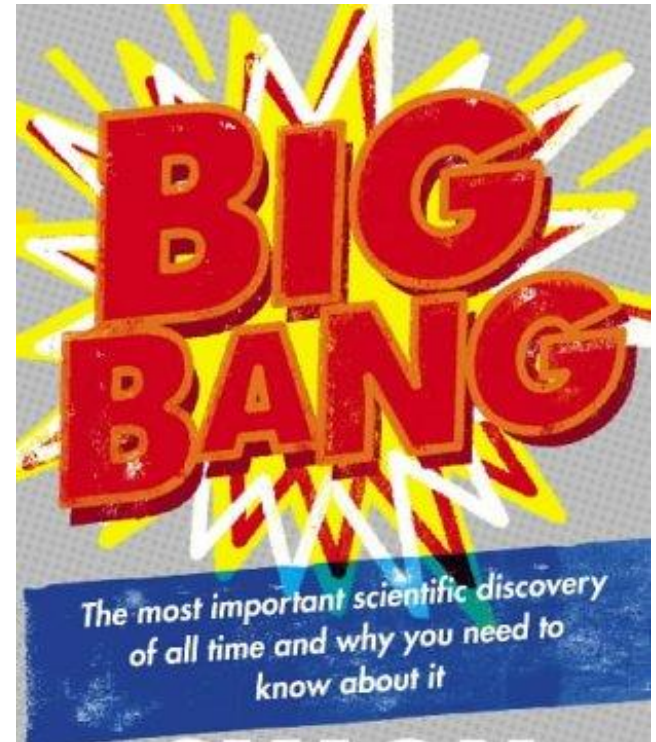
Evidence of a Hot Big Bang

Calculations of the amount of Helium expected to have been produced a few minutes just after the Big Bang agrees well with the amounts of He observed in primordial gas.

In order for the Universe to have produced those $H \rightarrow He$ fusion reactions it must have been extremely hot and dense filled with high energy photons.

As the universe expanded the plasma cooled and when it reached $T \sim 3000$ K electrons combined with protons to form neutral H. At this point ($\sim 380,000$ years after the Big Bang) photons were able to escape the plasma and travel freely through space.

Since then the universe has expanded by a factor of about 1100.



Radiation of the Universe

Photons are massless particles that travel across the Universe at the speed of light and constitute a form of radiation.

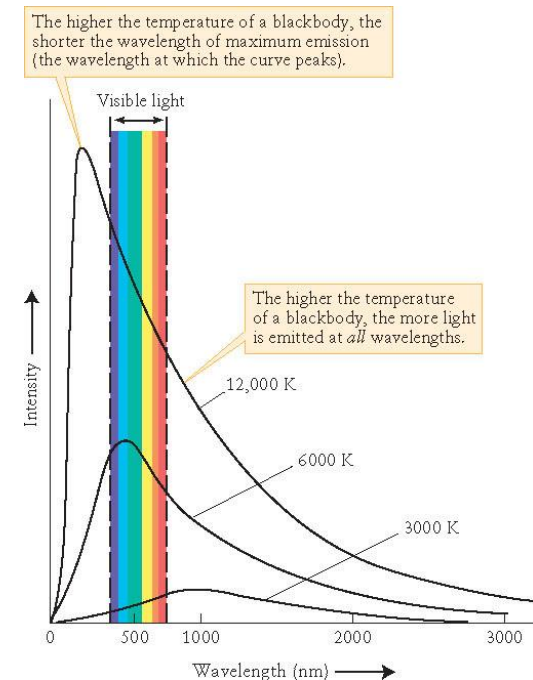
Radiation from the hot plasma produced just after the Big Bang has a **blackbody spectrum**.

Recall that a Blackbody spectrum is characterized by a peak wavelength and a Temperature that follow Wien's Law:

$$\lambda_{max}(m) = \frac{0.0029 \text{ K m}}{T(K)}$$

$\lambda_{max}(m)$ = wavelength of maximum emission in meters

T = temperature of object in kelvins



Evidence of a Hot Big Bang

In 1965 Arno Penzias and Robert Wilson while working at Bell Labs on a horn antenna discovered cosmic background radiation left over from the hot Big Bang.

Temperature of the cosmic microwave background (CMB) now $T_{\text{obs}} = 2.725 \text{ K}$

Because of the cosmological redshift the spectrum of the CMB is redshifted making the observed temperature now much cooler than the original one of 3,000 K!

$$T_{\text{obs}} = 0.0029\text{K}/\lambda_{\text{obs}}$$

$$\lambda_{\text{obs}}/\lambda_0 = (1+z) \rightarrow \lambda_{\text{obs}} = (1+z) \lambda_0 \sim 1100 \lambda_0$$

$$\lambda_0 = 0.0029/3,000\text{K}$$

$$T_{\text{obs}} = 3,000\text{K}/1100 \sim 2.73 \text{ K}$$

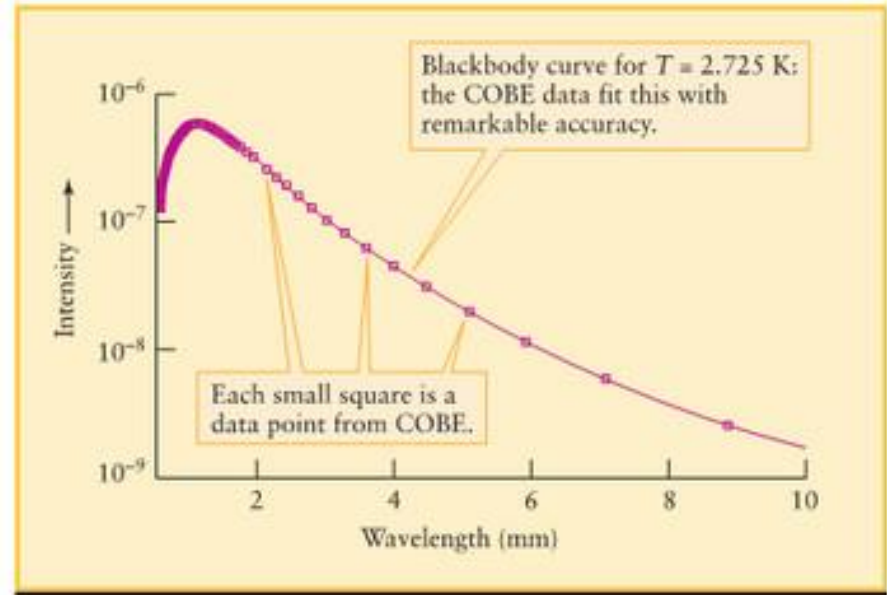


Penzias and Wilson in front of the Horn Antenna.

Evidence of a Hot Big Bang



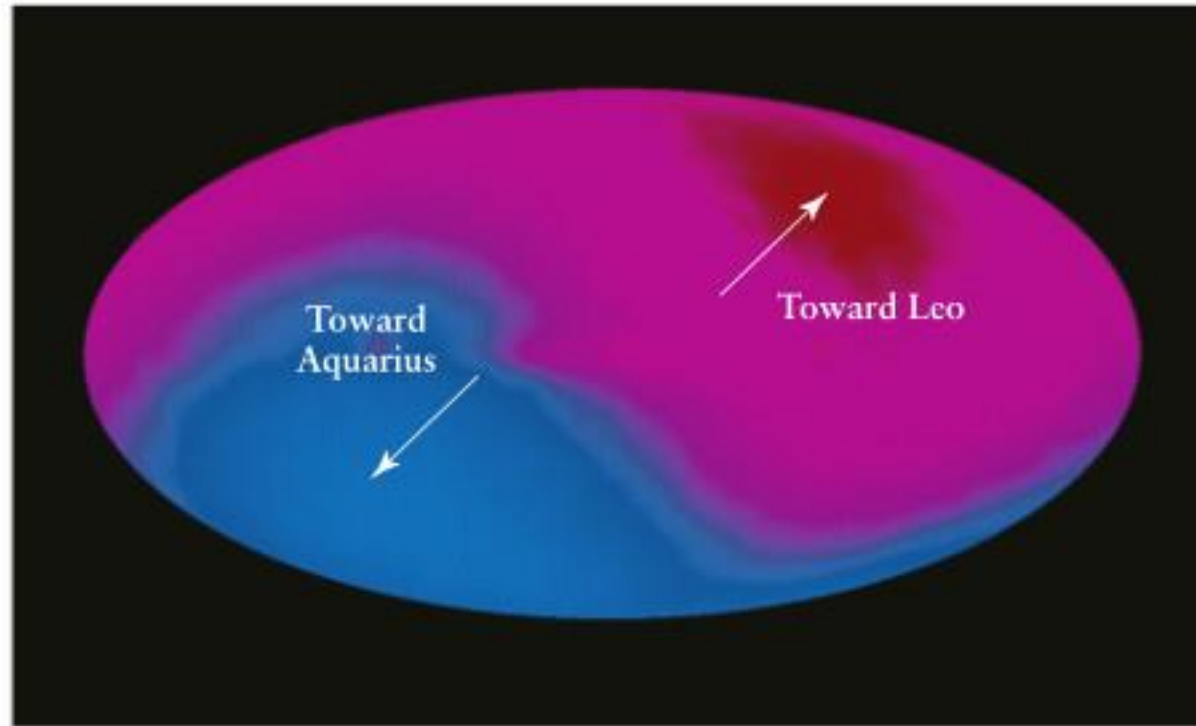
(a) The COBE spacecraft



(b) The spectrum of the cosmic microwave background

The first high-precision measurements of the cosmic microwave background came from the Cosmic Background Explorer (COBE) satellite, which was placed in Earth orbit in 1989. The CMB intensity is almost perfectly isotropic with a slight variation in temperature across the sky.

Evidence of a Hot Big Bang

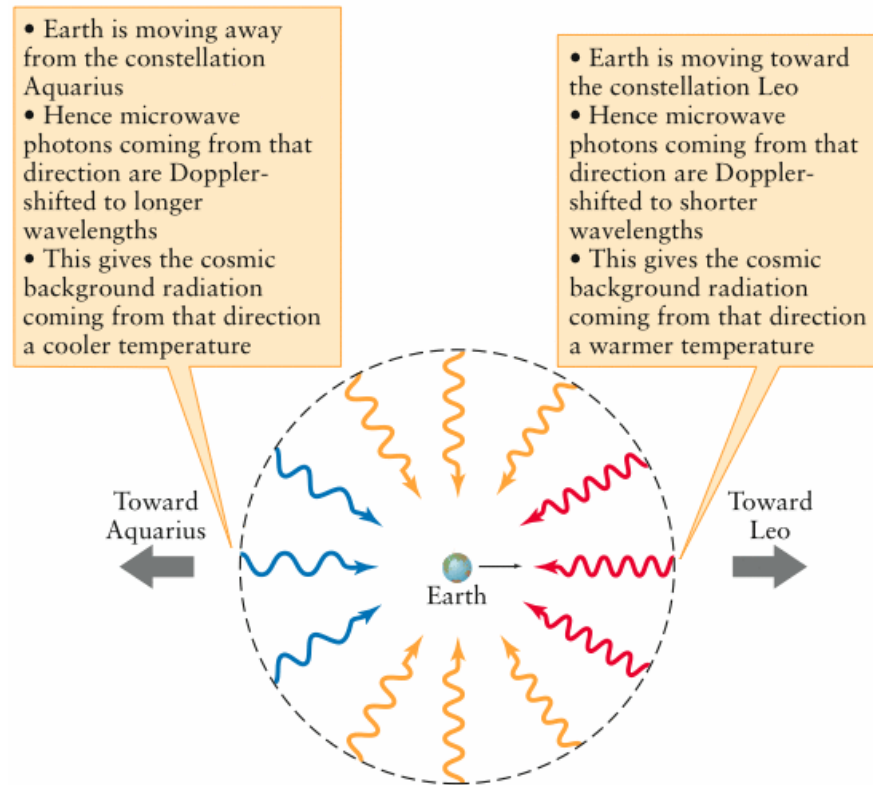


The microwave background appears slightly warmer than average toward the constellation of Leo and slightly cooler than average in the opposite direction toward Aquarius. In this map of the entire sky made from COBE data, the plane of the Milky Way runs horizontally across the map, with the galactic center in the middle. Color indicates temperature—red is warm and blue is cool. The small temperature variation across the sky is caused by Earth's motion through the microwave background.

Evidence of a Hot Big Bang

The Earth is moving through the CMB radiation with a peculiar velocity v .

Due to the Doppler effect the temperature of the CMB will depend on the direction of observation.

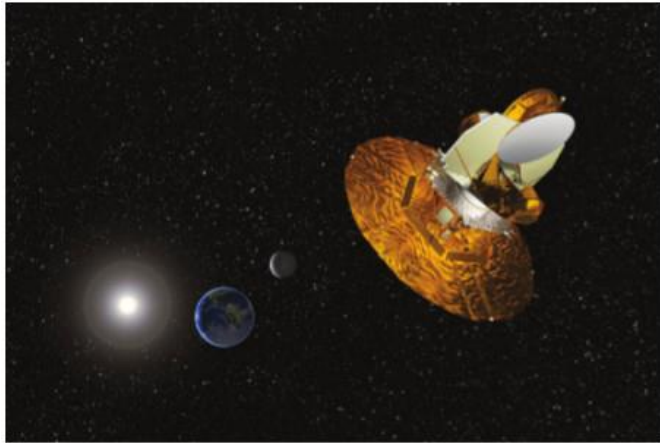


For $v \ll c$

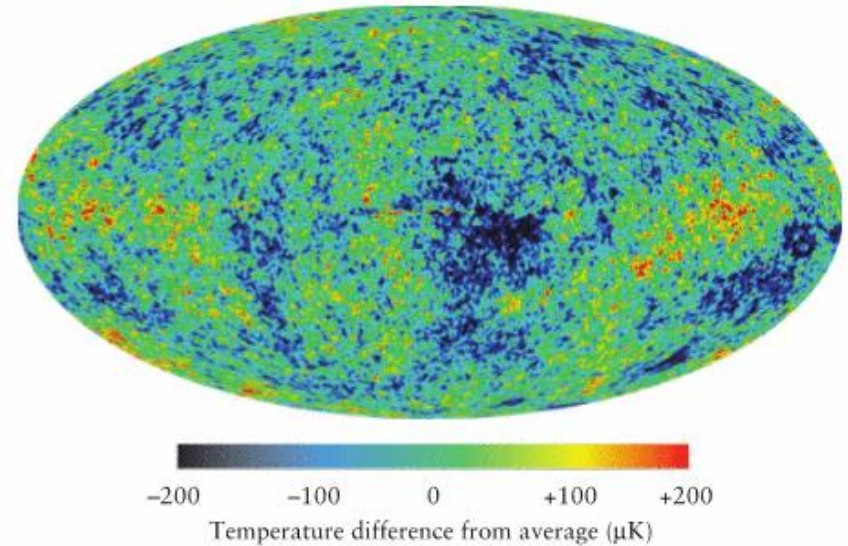
$$T_{obs} = T_{rest} \left(1 + \frac{v}{c} \cos \theta \right)$$

θ is the angle between the direction of observation and the velocity of Earth relative to the CMB.

Temperature Variations in the CMB



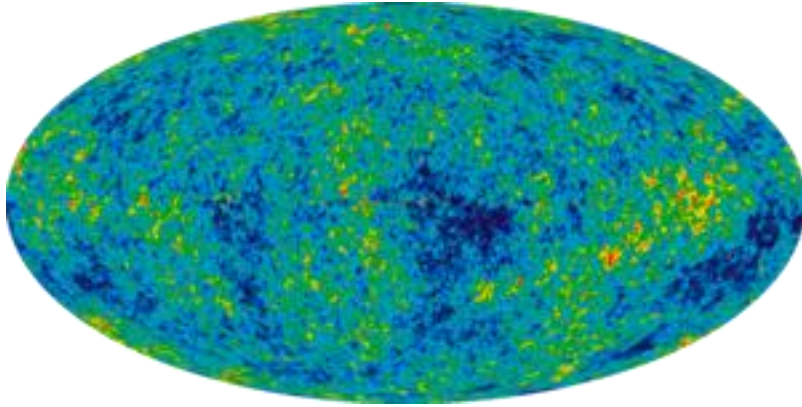
The Wilkinson Microwave Anisotropy Probe (WMAP) launched in 2001.



This map from WMAP data shows small variations in the temperature of the cosmic background radiation across the entire sky.

WMAP images show temperature variations of the CMB of the order of 2×10^{-4} K around the average value of 2.725K. The denser regions will evolve to become the galaxies and clusters of galaxies in the Universe. This 2.725K background radiation originates from the recombination era some $\sim 380,000$ years after the Big Bang.

Cosmic Microwave Background



When H atoms first formed 380,000 years after the Big Bang light was able to freely travel through the Universe.

Temperature of CMB Then $\sim 3,000\text{K}$

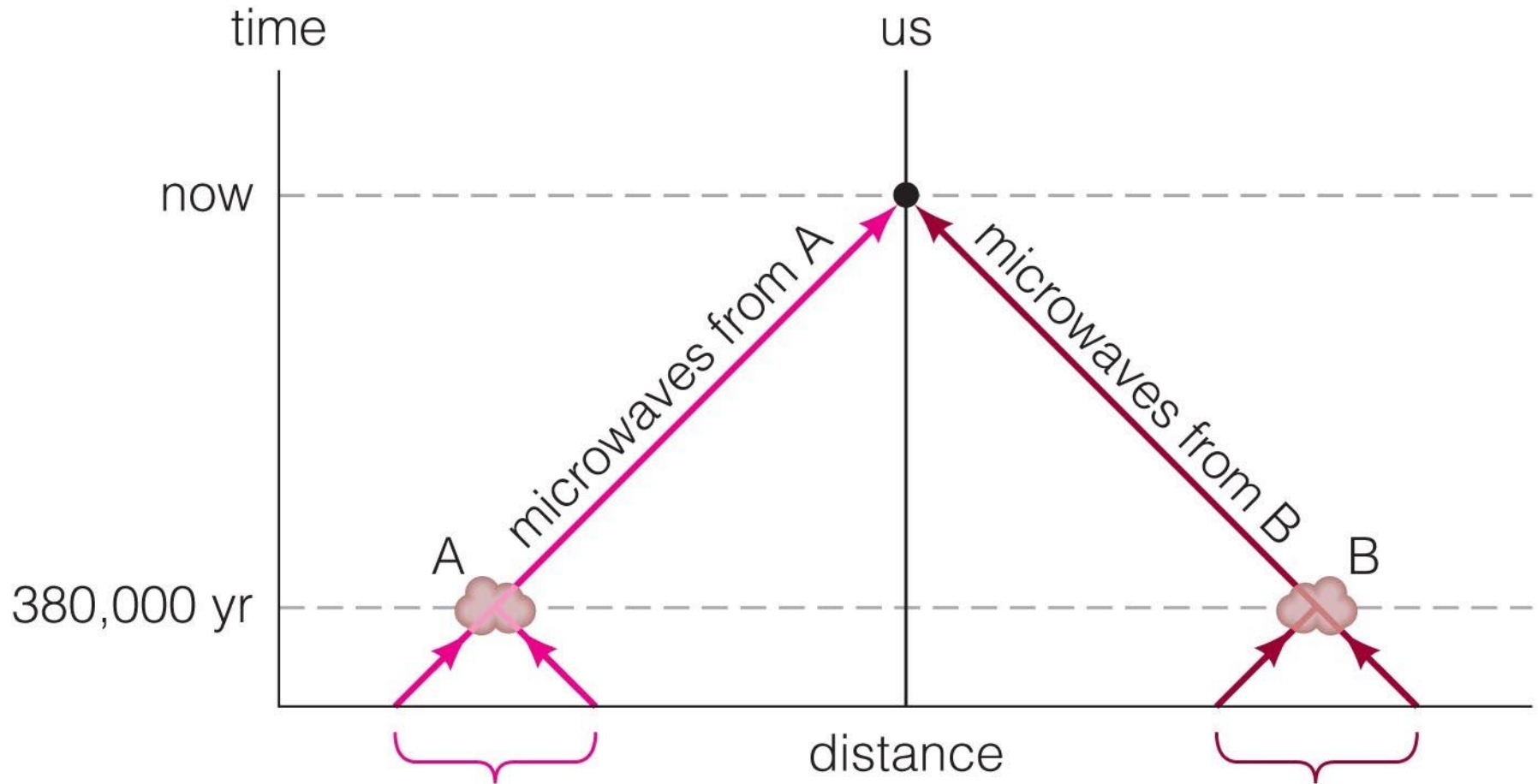
Temperature of CMB NOW $\sim 2.725\text{ K}$

A great discovery that confirmed the big bang theory was the detection of the cosmic microwave background (CMB).

A careful analysis of the CMB revealed two strange results:

1. It's too uniform (or isotropic).
2. The curvature of space of the Universe is near zero today.

Result 1: Universe is uniform over large scales



- How can microwave temperature be nearly identical on opposite sides of the sky?

Result 2: The Early Universe: The Flatness Problem

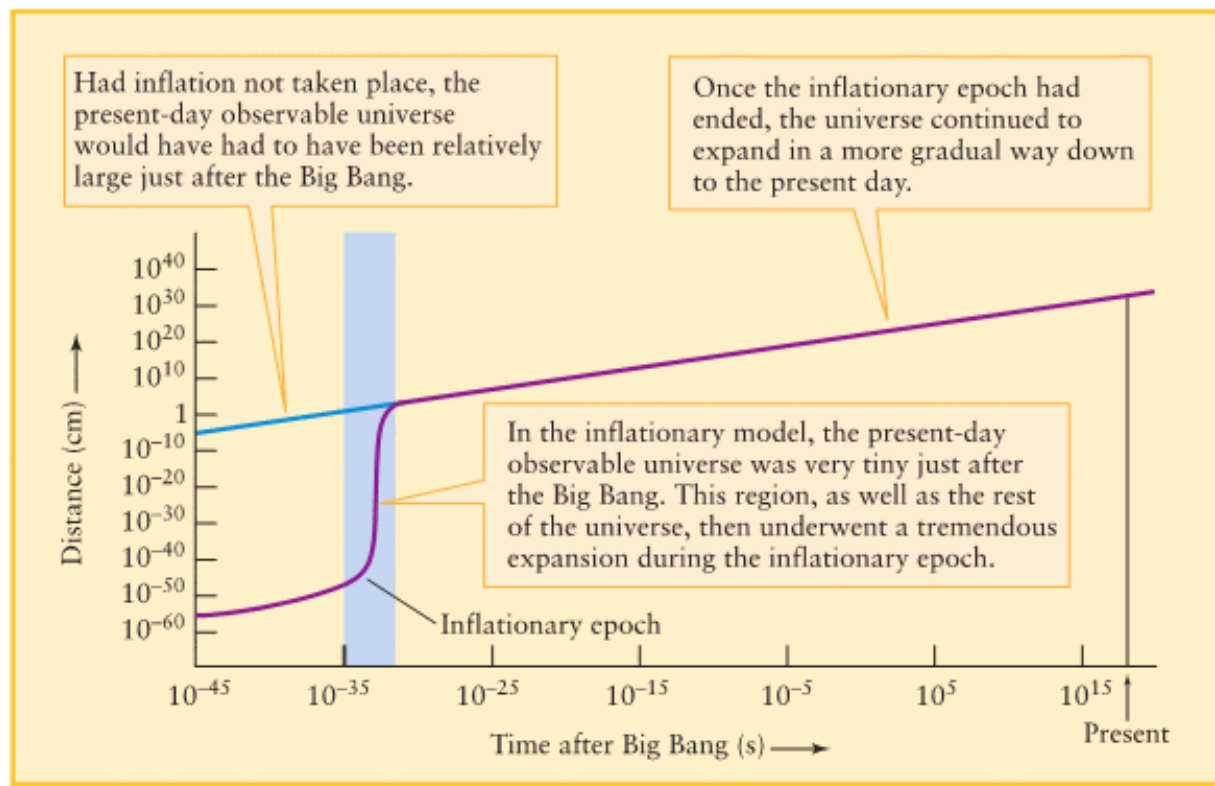
The geometry of our universe depends on the density parameter Ω_0 , which is the ratio of the combined mass density in the universe (ρ_0) to the critical density (ρ_c)

$$\Omega_0 = \rho_0 / \rho_c$$

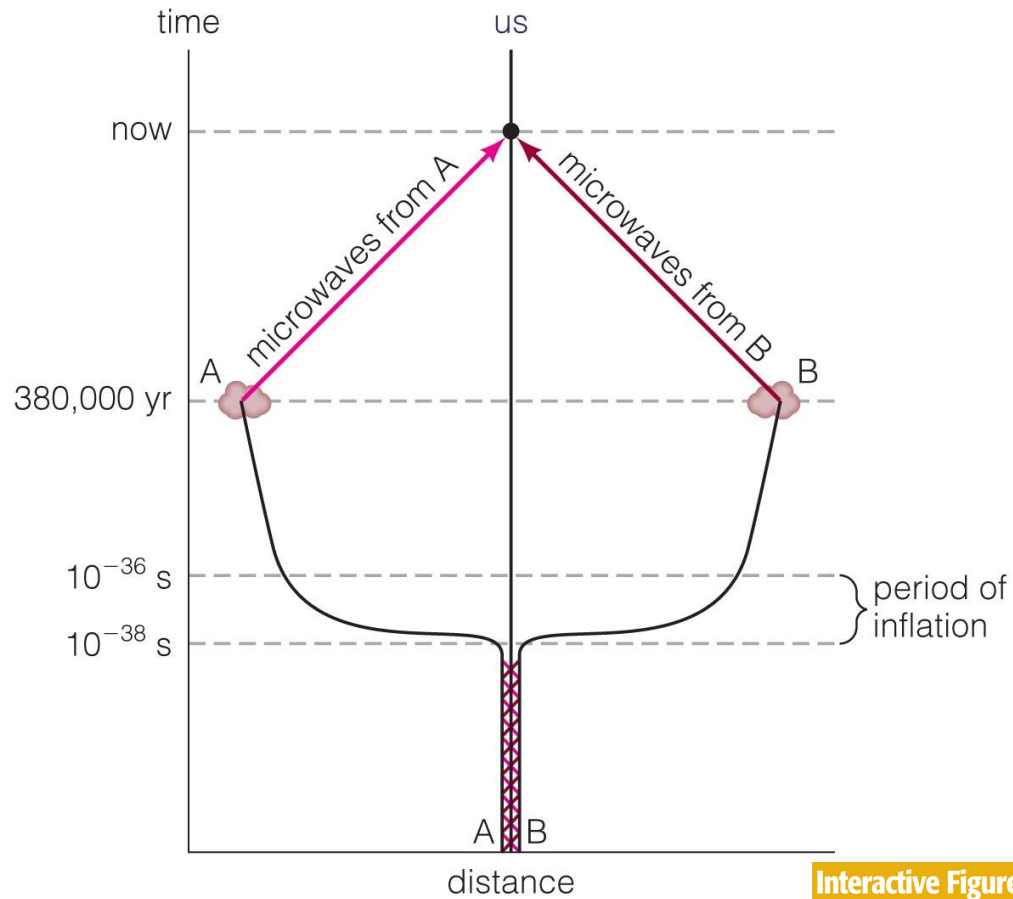
We now know that the density of the Universe is exactly equal to the critical density: $\Omega_0=1$ (flat Universe).

Proposed solution to results (1) and (2): Inflation

Independently Alexei Starobinsky and Alan Guth suggested that the universe might have experienced a brief period of inflation, expanded by a factor of 10^{50} , shortly after the big bang.

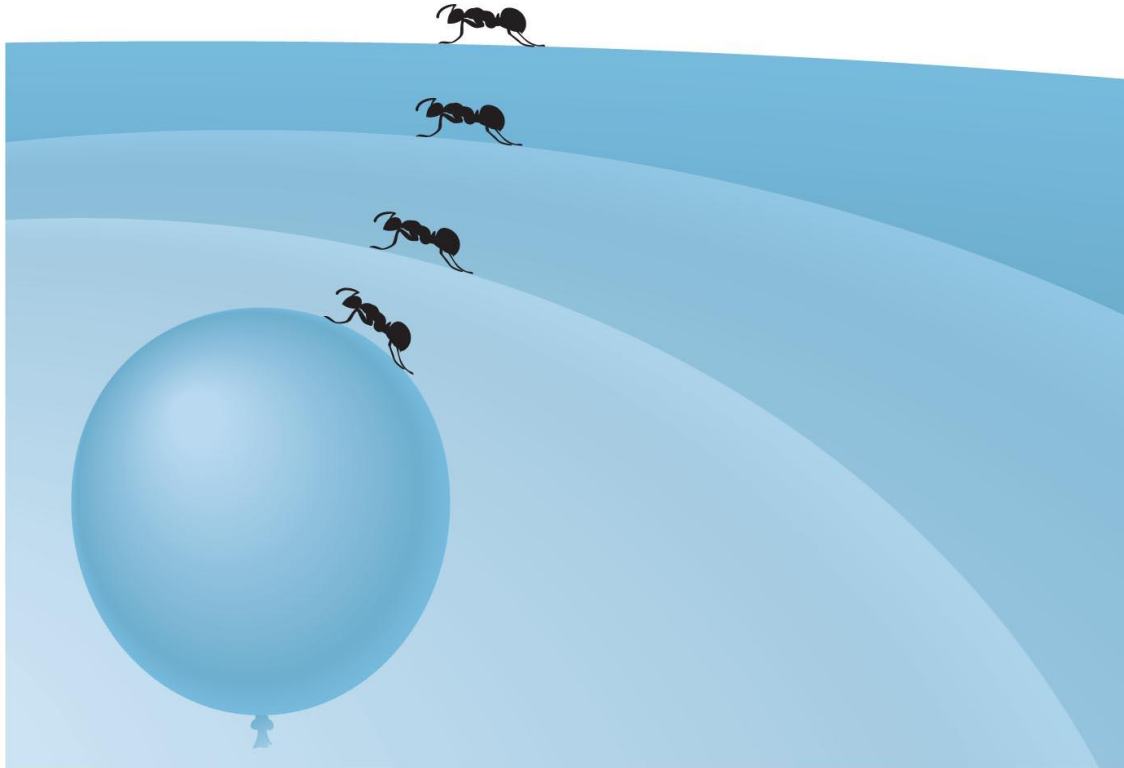


Inflation explains uniformity of Universe



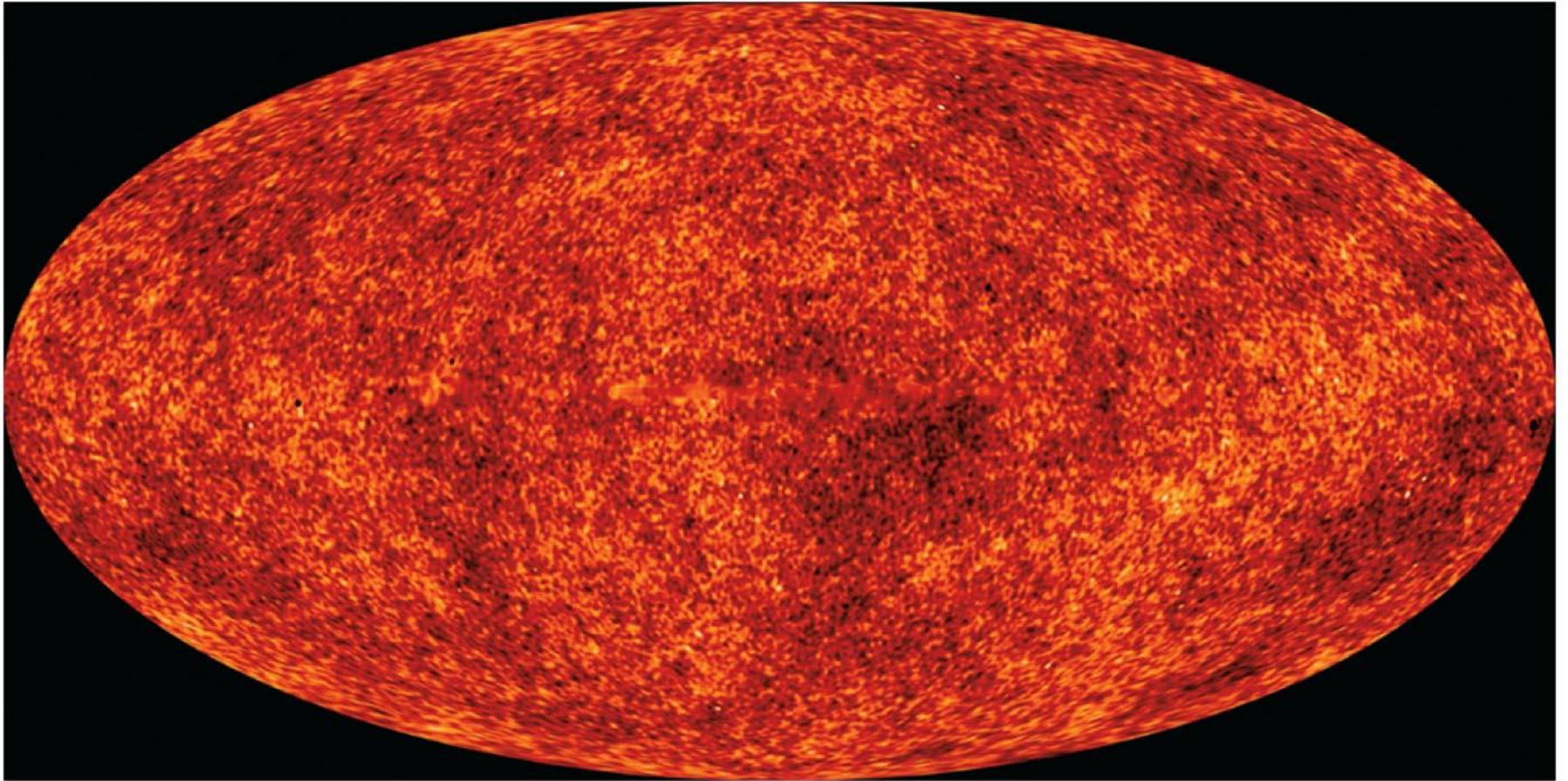
- Regions now on opposite sides of the sky were close together before inflation pushed them far apart.

Inflation explains flatness of Universe

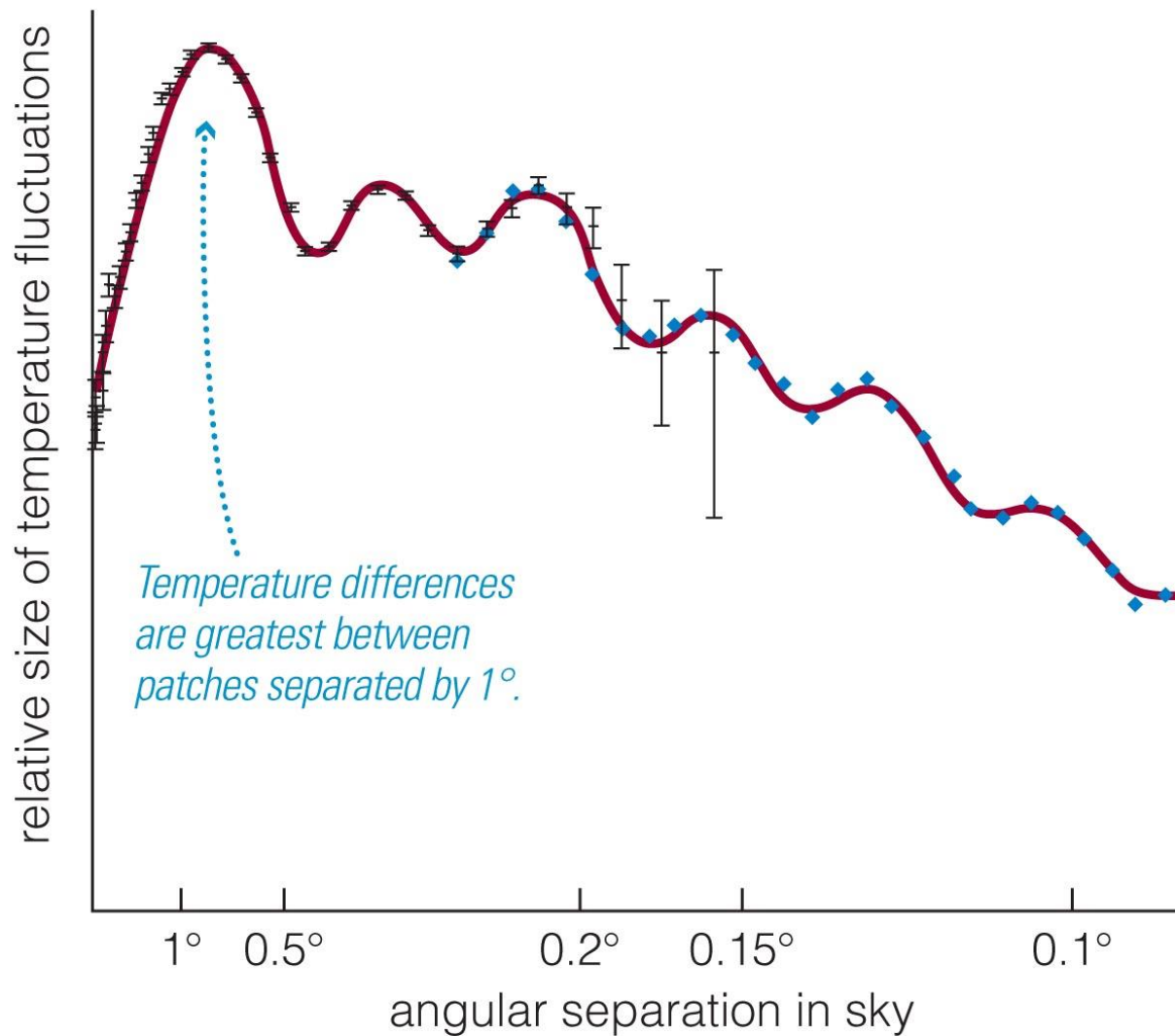


- Inflation of the universe flattens its overall geometry like the inflation of a balloon, causing the overall density of matter plus energy to be very close to the critical density.

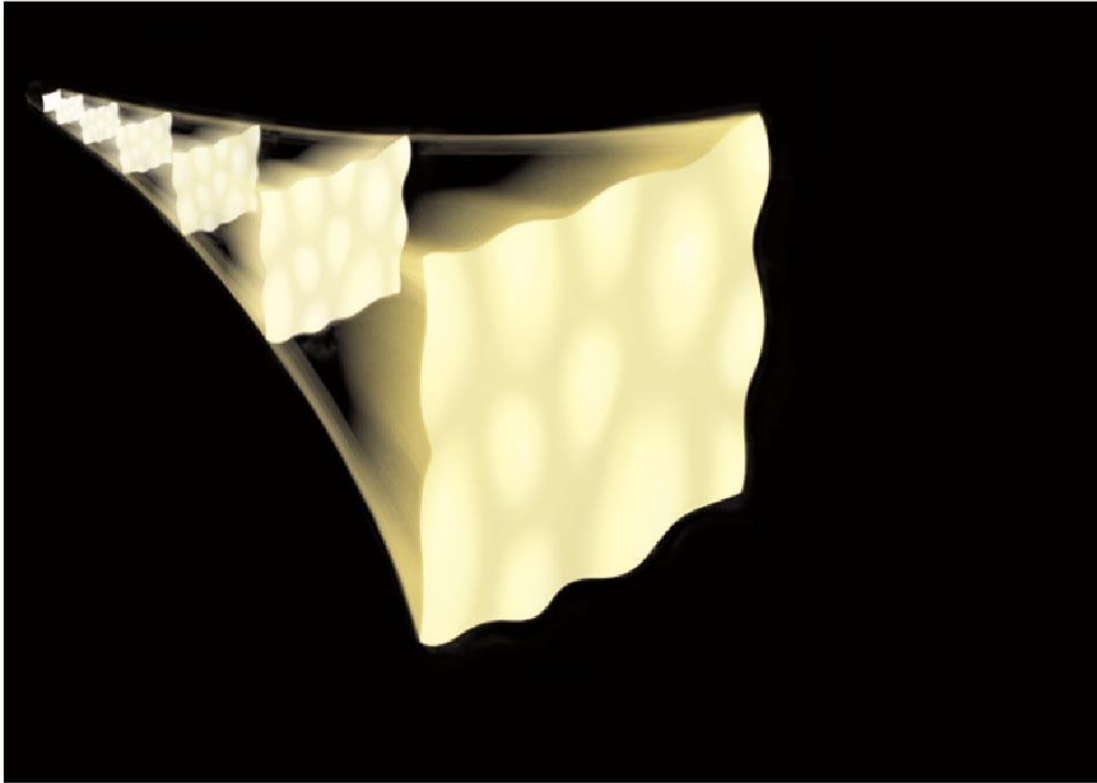
Did inflation really occur?



- ▣ Patterns observed by WMAP show us the "seeds" of structure in the universe.



- Observed patterns of structure in universe agree (so far) with the "seeds" that inflation would produce.



- Inflation can make all the structure by stretching tiny quantum ripples to enormous size.
- These ripples in density then become the seeds for all structures in the universe

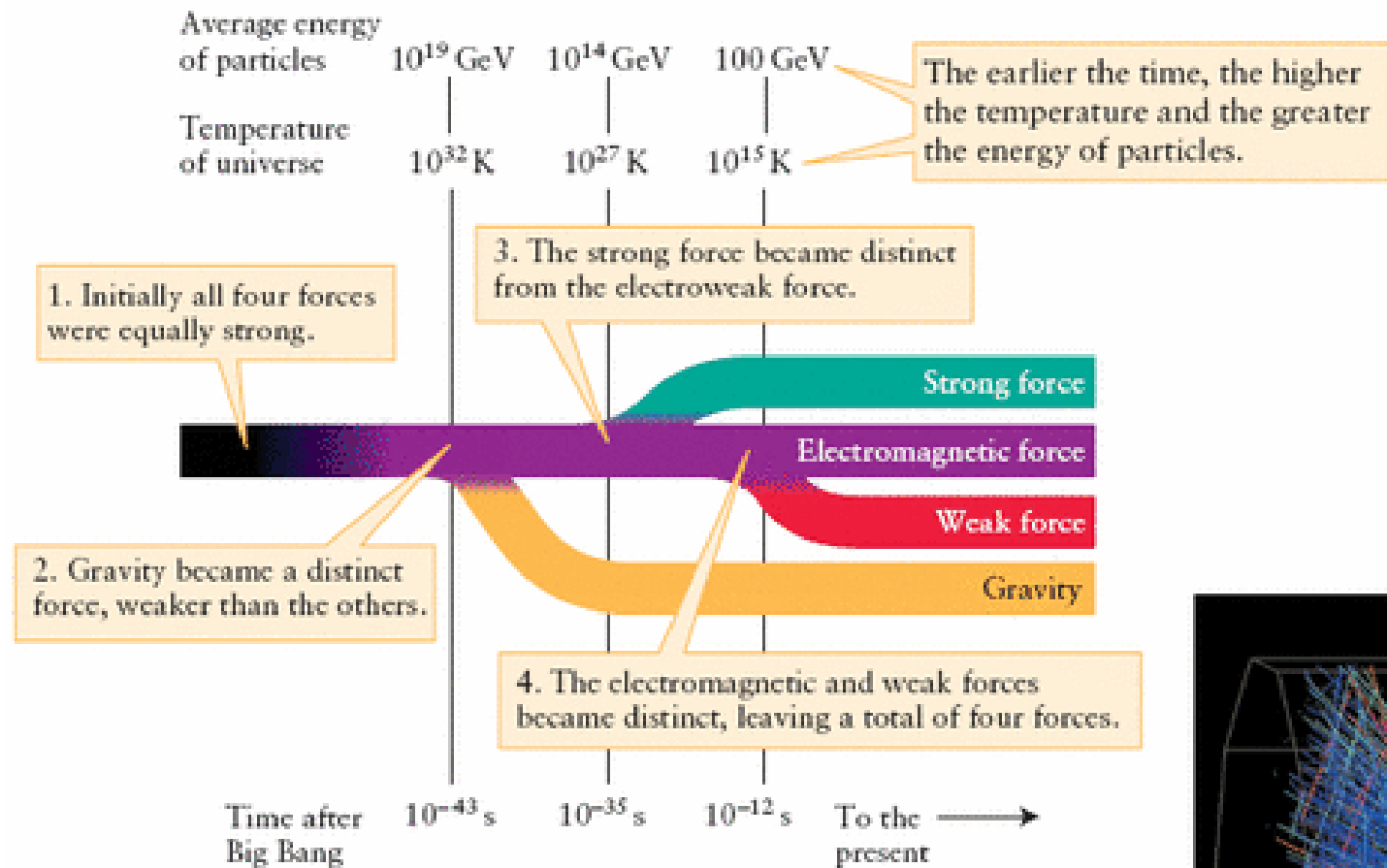
The Early Universe: Fundamental Forces

To understand what happened in the early universe we need to understand how particles interact at high energies.

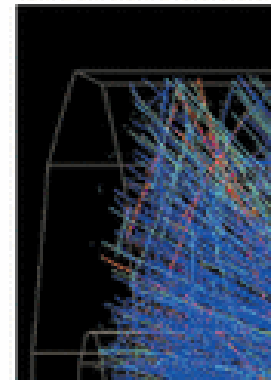
Particles interact through **four fundamental forces of nature**.

1. **Gravity** (force between everything including massless objects like photons, infinite range)
2. **Electromagnetism** (force between charged particles, infinite range)
3. **Strong Force** (force between quarks that make up neutrons and protons, short-range force $\sim 10^{-15}\text{m}$)
4. **Weak Force** (force acts on quarks, electrons, neutrinos and involved in certain radioactive decays, short-range force $\sim 10^{-16}\text{m}$)

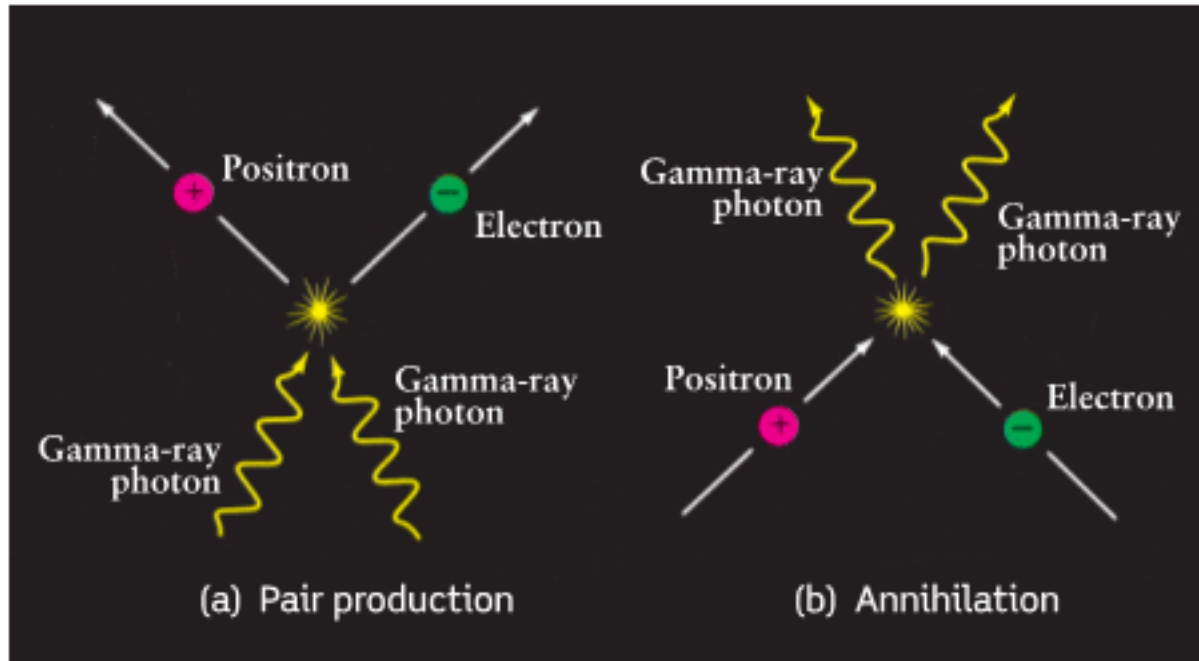
One Force in the Beginning?



(a) How the four forces behave at different energies and temperatures



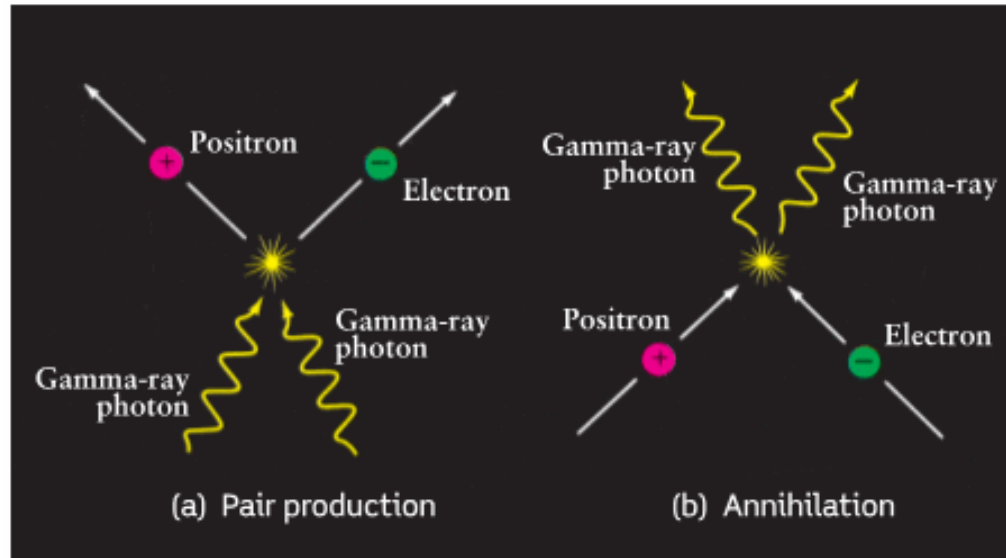
Matter and Radiation Created During Inflation



Pair production can take place only if the combined energy of the two photons is greater than Mc^2 , where M is the total mass of the electron and positron

In the Early Universe the rate of pair production was equal to the rate of annihilation but as the Universe expanded and the temperature dropped...

Matter and Radiation Created During Inflation



The rate of pair production decreased as the Universe expanded but annihilation continued.

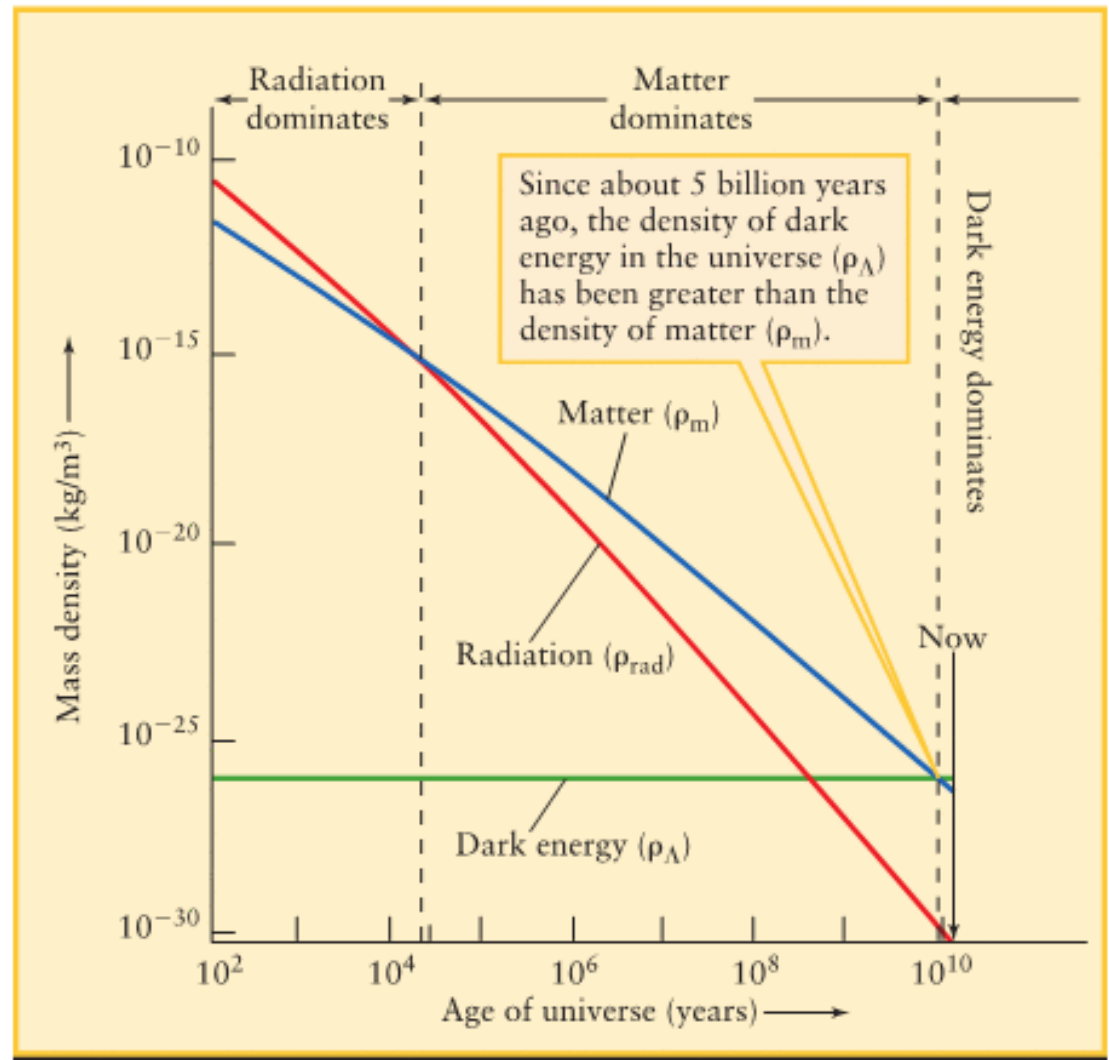
In the present Universe we mostly find matter and not antimatter. But why did not all matter and antimatter annihilate?

Theories of elementary particles predict an excess of matter over antimatter immediately after the Big Bang, so that the particles outnumbered the antiparticles.

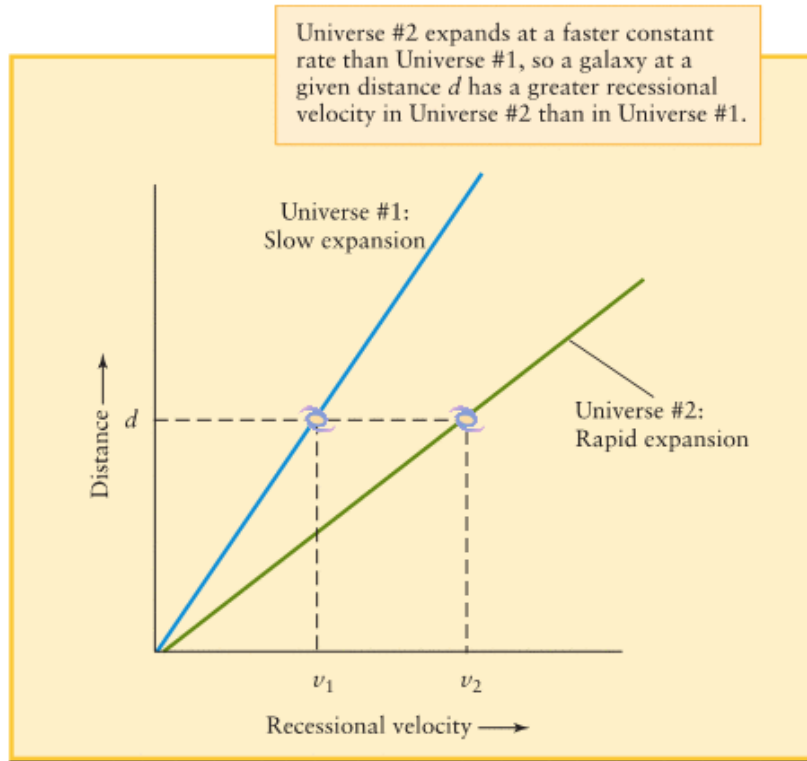
Why?

In the past dark energy was unimportant and in the future it will be dominant!

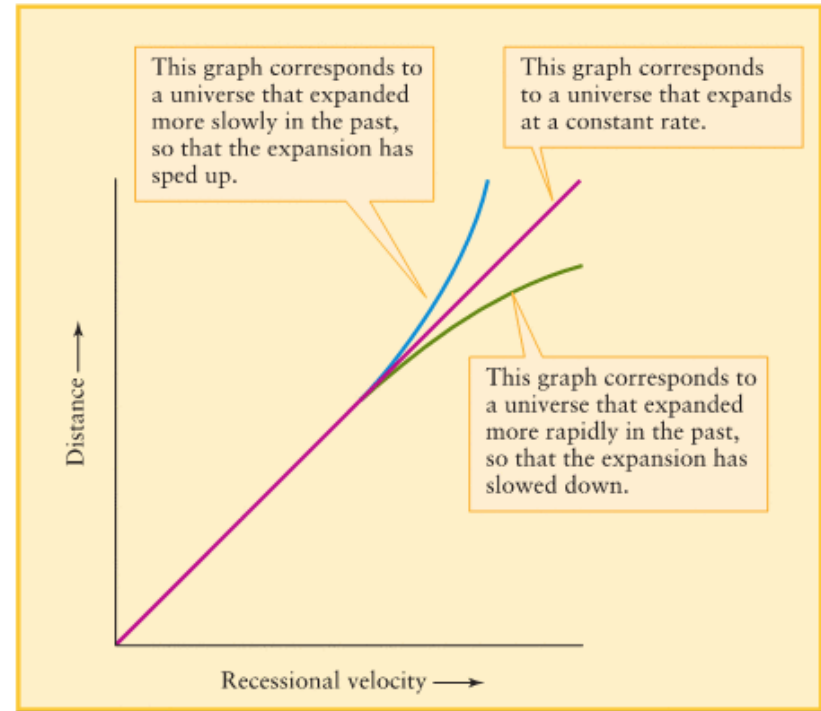
We just happen to live at the time when dark matter and dark energy have comparable densities.



Does The Expansion Rate Change With Time?



(a) Two universes with different expansion rates



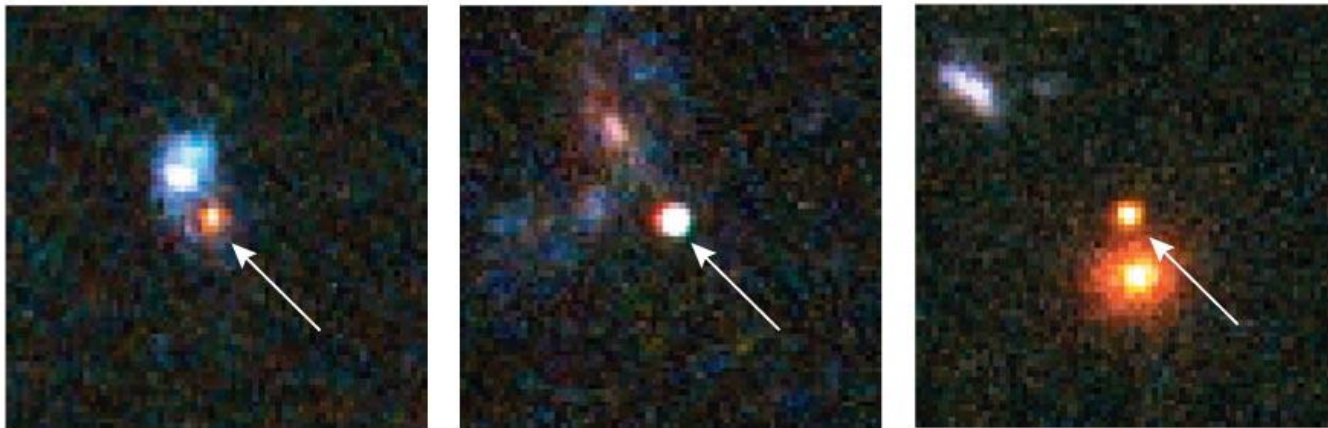
(b) Possible expansion histories of the universe

To address this question we need to look at the distances versus recession velocities of objects and see if the expansion rate changes with redshift. If the expansion rate is slowing down we expect a a steeper slope in the Distance versus velocity plot.

Distant galaxies before supernova explosions

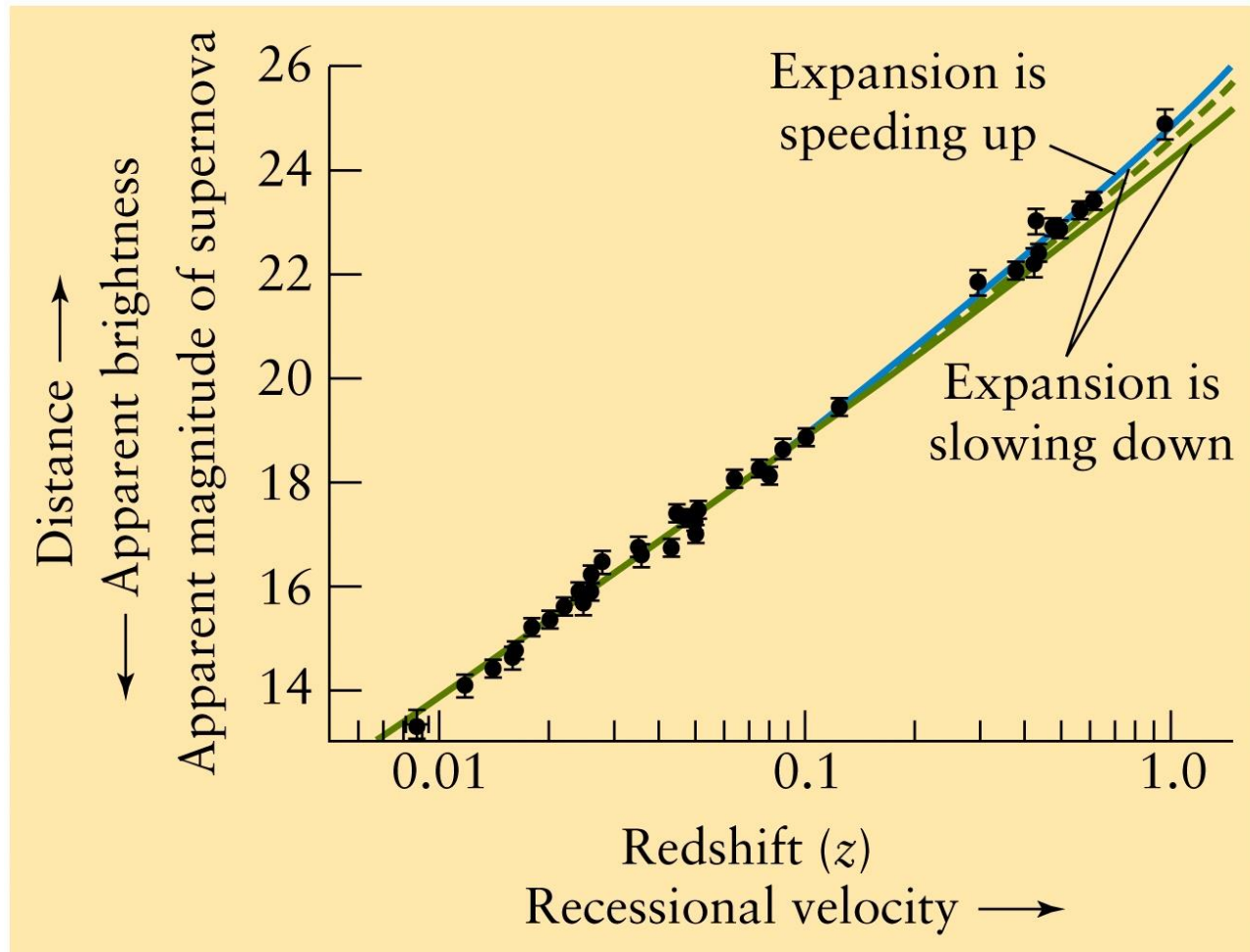


The same galaxies after supernova explosions



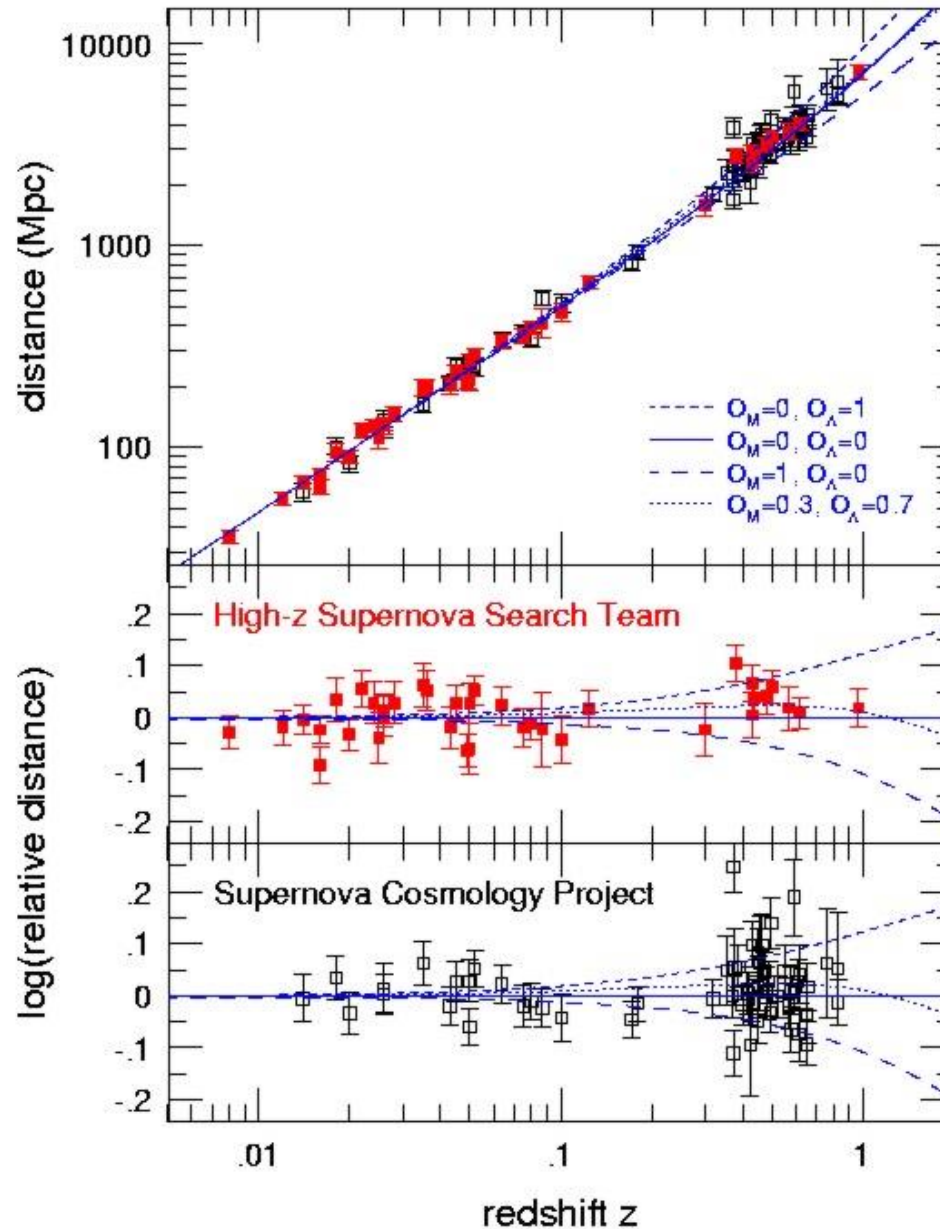
- The brightness of distant white dwarf supernovae tells us how much the universe has expanded since they exploded.

Does The Expansion Rate Change With Time?



The data from SN Ia follow the blue curve and show that the Universe was expanding at a slower rate in the past. The expansion of the Universe is now speeding up!

Does The Expansion Rate Change With Time?



Friedmann Equation

The presence of the cosmological constant Λ was indicated by observations in the late 1990's of very distant SN type Ia. They showed that the expansion of the Universe was accelerating. The source of the acceleration is called Dark Energy. The **Friedmann equation**, including the cosmological constant is:

$$\left[\left(\frac{1}{R} \frac{dR}{dt} \right)^2 - \frac{8}{3} \pi G (\rho_m + \rho_{rel}) - \frac{1}{3} \Lambda c^2 \right] R^2 = -k c^2$$
$$\left[\left(\frac{1}{R} \frac{dR}{dt} \right)^2 - \frac{8}{3} \pi G (\rho_m + \rho_{rel} + \rho_\Lambda) \right] R^2 = -k c^2$$

Where $R(t)$ is the scale factor, and ρ_m , ρ_{rel} , and ρ_Λ are the densities of matter (baryonic and dark), relativistic particles and dark energy, respectively.

Where $\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}$ is the density of dark energy.

In the Friedmann equation, k represents the curvature of the Universe. In the Newtonian formalism k represents the total energy of the Univers

The Fluid Equation is: $\frac{dR^3 \rho}{dt} = -\frac{P}{c^2} \frac{dR^3}{dt}$

Acceleration Equation

$$\frac{d^2 R}{dt^2} = \left\{ -\frac{4}{3} \pi G \left[\rho_m + \rho_{rel} + \rho_\Lambda + \frac{3(P_m + P_{rel} + P_\Lambda)}{c^2} \right] \right\} R$$

Where $\rho_\Lambda = \frac{\Lambda c^2}{8\pi G}$ and the pressure due the dark energy is $P_\Lambda = -\rho_\Lambda c^2$

Since $P = wu = w\rho c^2$ the Fluid Equation can also be written as:

$$R^{3(w+1)} \rho = \rho_0$$

For matter $w = 0 \rightarrow R^3 \rho_m = \rho_{m,0} \rightarrow \rho_m = \rho_{m,0} (1+z)^{-3}$

For relativistic particles $w = 1/3 \rightarrow R^4 \rho_{rel} = \rho_{rel,0} \rightarrow \rho_{rel} = \rho_{rel,0} (1+z)^{-4}$

For dark energy $w = -1 \rightarrow \rho_\Lambda = \rho_{\Lambda,0}$ (dark energy density remains constant)

w is called the equation of state parameter.

Critical Density and Curvature of Universe

The critical density (which leads to zero curvature) of the Universe is:

$$\rho_c(t) = \frac{3H^2(t)}{8\pi G}$$

The density parameters are defined as:

$$\Omega_m(t) = \frac{\rho_m(t)}{\rho_c(t)} = \frac{\rho_m(t)8\pi G}{3H^2(t)}, \Omega_{rel}(t) = \frac{\rho_{rel}(t)}{\rho_c(t)} = \frac{\rho_{rel}(t)8\pi G}{3H^2(t)}, \Omega_\Lambda(t) = \frac{\rho_\Lambda(t)}{\rho_c(t)} = \frac{\rho_\Lambda(t)8\pi G}{3H^2(t)}$$

Recall that $v(t) = H(t)r(t) \rightarrow \frac{dr(t)}{dt} = H(t)r(t) \rightarrow \frac{dR(t)}{dt} = H(t)R(t) \rightarrow H(t) = \frac{1}{R(t)} \frac{dR(t)}{dt}$

The Friedmann Equation becomes:

$$H(t)^2 [1 - (\Omega_m(t) + \Omega_{rel}(t) + \Omega_\Lambda(t))] R^2 = -kc^2$$

For a Universe with $k=0$ (zero curvature) $\Omega_m(t) + \Omega_{rel}(t) + \Omega_\Lambda(t) = 1$

For $k>0 \rightarrow (\Omega_m(t) + \Omega_{rel}(t) + \Omega_\Lambda(t)) > 1$ this leads to a bound/closed Universe

For $k<0 \rightarrow (\Omega_m(t) + \Omega_{rel}(t) + \Omega_\Lambda(t)) < 1$ this leads to an open Universe

WMAP RESULTS

$$\Omega_{m,0} = 0.27 \pm 0.04$$

$$\Omega_{rel,0} = 8.24 \times 10^{-5}$$

$$\Omega_{\Lambda,0} = 0.73 \pm 0.04$$

$$\Omega_0 = 1.02 \pm 0.02$$

WMAP finds that the total density parameter of the Universe today is consistent with 1. The WMAP results show that the Universe has zero curvature.

The Hubble Parameter $H(z)$ is:

$$H(z) = H_0(1+z) \left[\Omega_{m,0}(1+z) + \Omega_{rel,0}(1+z)^2 + \frac{\Omega_{\Lambda,0}}{(1+z)^2} + 1 - \Omega_0 \right]^{1/2}$$

Evolution of Mass Density

The FLUID EQUATIONS for matter, relativistic particles, and dark energy:

$$R^{3(w+1)}\rho = \rho_0$$

For matter $w = 0 \rightarrow R^3 \rho_m = \rho_{m,0} \rightarrow \rho_m = \rho_{m,0}(1+z)^3$

For relativistic particles $w = 1/3 \rightarrow R^4 \rho_{rel} = \rho_{rel,0} \rightarrow \rho_{rel} = \rho_{rel,0}(1+z)^4$

For dark energy $w = -1 \rightarrow \rho_\Lambda = \rho_{\Lambda,0}$ (dark energy density remains constant)

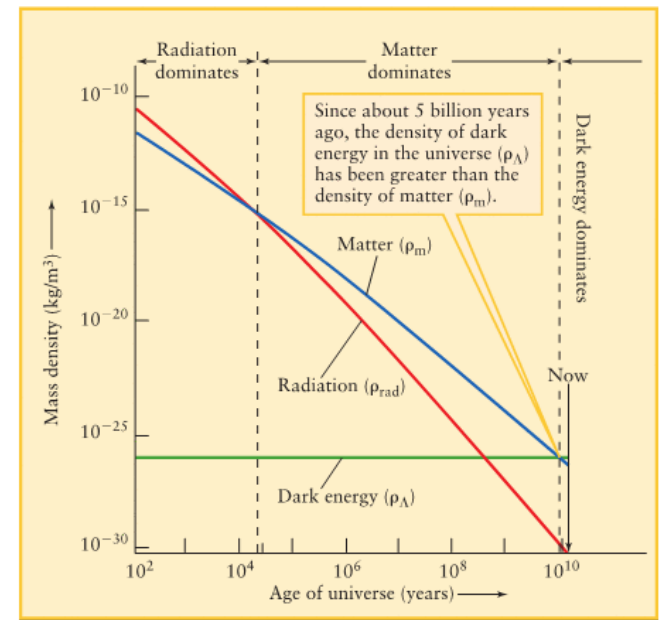
Example: At what redshift did the matter density of the Universe equal the dark energy density?

Set $\rho_m = \rho_\Lambda$

$$\rho_m = \rho_{m,0} R^{-3} = \rho_\Lambda = \rho_{\Lambda,0}$$

Divide by ρ_c to obtain density parameters.

$$\Omega_{m,0} R^{-3} = \Omega_{\Lambda,0} \rightarrow R^{-3} = \left(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}} \right) \rightarrow R = \left(\frac{\Omega_{\Lambda,0}}{\Omega_{m,0}} \right)^{1/3}$$



Using the WMAP results we obtain the scale factor at which $\rho_m = \rho_\Lambda$ to be

$$R = 0.72 \rightarrow \frac{1}{1+z} = 0.72 \rightarrow z = 0.39$$

Cooling of CMB Radiation

The equation of state is: $P = wu = w\rho c^2$

The fluid equation is: $R^{3(w+1)}\rho = \rho_0$

For CMB blackbody radiation $w = \frac{1}{3}$ and $u = aT^4$

$$u = \rho c^2 \rightarrow \rho_{rad} = \frac{u}{c^2} = \frac{aT^4}{c^2}$$

The fluid equation of the CMB radiation becomes $R^4 \rho_{rad} = \rho_{rad,0} \rightarrow$

$$R^4 \frac{aT_{emit}^4}{c^2} = \frac{aT_0^4}{c^2} \rightarrow R^4 T_{emit}^4 = T_0^4 \rightarrow RT_{emit} = T_0 \rightarrow T_0 = \frac{T_{emit}}{(1+z)}$$

The temperature of the CMB now is about $T_0 \approx 2.73K$ and the temperature when it was emitted was $T_{emit} \approx 3000K$

Evolution of Scale Factor

The evolution of the scale factor for a flat Universe is found by solving the Friedmann equation:

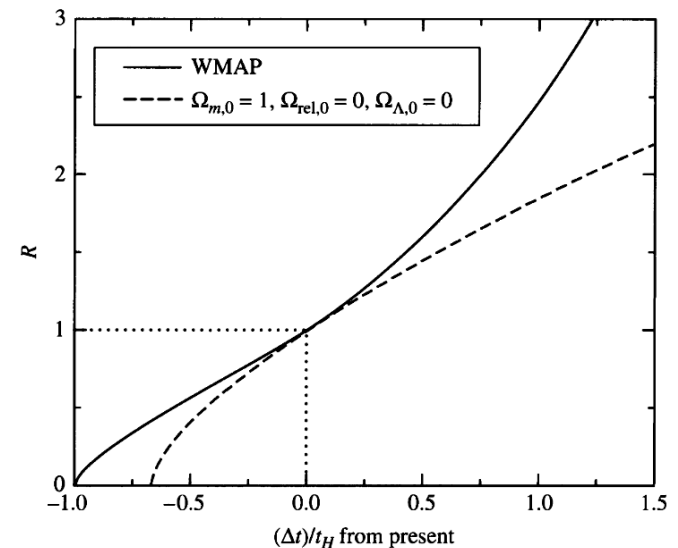
$$\left[\left(\frac{1}{R} \frac{dR}{dt} \right)^2 - \frac{8}{3} \pi G (\rho_m + \rho_{rel} + \rho_\Lambda) \right] R^2 = 0$$

For the radiation era: $R(t) \propto t^{1/2}$

For the matter dominated era: $R(t) \propto t^{2/3}$

For the dark energy dominated era the Universe grows exponentially:

$$R(t) = \left(\frac{\Omega_{m,0}}{4\Omega_{\Lambda,0}} \right)^{1/2} e^{H_0 t} \sqrt{\Omega_{\Lambda,0}}$$



EXTRA SLIDES

